01 A game to get things going

Put your team name and the name of your team members and your section number on the worksheet you intend to hand in.

Teams 1, 4, 7: Play this game and record your results

Rule 1: There are three players, Greedy, Fair, and Giving. (If you can think of a word starting with a hard "G" that means "fair," you get a gold star.) At the beginning of the game, each player has the same number of pennies. We'll start everyone off with 20 pennies.

Rule 2: The play proceeds in turns, and every player takes her or his turn at the same time. During each turn, each player divides her or his pennies into three piles of specific sizes, keeps one of those piles, and gives one to each of the other players.

(a) Greedy keeps 3/5 of her or his pennies and gives 1/5 to each of the other players.

(b) Fair keeps 1/3 of her or his pennies and gives 1/3 to each of the other players.

(c) Giving keeps 1/5 of her or his pennies and gives 2/5 to each of the other players.

Rule 3: In case the number of pennies is not evenly divisible by 5, Greedy will round up for her or his pile and down for the others', while Giving will round down for her or his pile and round up for the others'. True to the name, Fair will always be sure to give the other two players the same number of pennies, as close to 1/3 of the total as possible. (Why can this always be done?)

Rule 4: Play continues until all three players are bored!

Play this game for a while. Below are some questions to think about as you're playing. Please jot your answers and thoughts down in the space provided.

1. What do you notice about the number of pennies each player has after you've played a fairly large number (*i.e.*, 10 or 20) of turns? How many pennies does each player have?

[1]These materials are inspired by some created by Patrick Bahls of UNC Asheville. All errors are my own.

2. We will spend some of our time this semester analyzing this game and a large number of other mathematical applications like it. For now, we don't know enough to do a lot of analysis, but let's start applying some mathematical ideas to describe what is going on.

   Let's let x_n, y_n, and z_n denote the number of pennies that Greedy, Fair, and Giving have, respectively, after n turns have been taken.

   (a) What's x_0, y_0 and z_0?

   (b) Going back to the original rules of the game, find a formula for x_1 in terms of x_0, y_0, and z_0.

   (c) Do the same as you did in (b) for y_1 and z_1.

   (d) There's really nothing special about $n = 1$: let's just keep n variable and find formulas for x_{n+1}, y_{n+1}, and z_{n+1} in terms of x_n, y_n, and z_n:

(e) **Time Travel.** Suppose that we're playing our "Greedy/Giving/Fair" game and after a certain turn has taken place, we see that Greedy has 30 pennies, Fair has 25, and Giving has 15. Your job: how would you determine how many pennies each player had *before* the last turn was played. (*Hint:* think of the current distribution of pennies as x_{n+1}, y_{n+1}, and z_{n+1}. What are you trying to find?) Describe what you would do, and, if you can, use a computer or calculator to help you find a solution.

The relationships we've derived between each x_n, y_n, and z_n and the same values in the previous turn collectively give us our first example of a **system of linear equations**. Obviously each expression is an **equation**; their interdependence makes the collection a **system** of equations, and each of them is **linear** because none of them involve powers of variables higher than 1.

We will spend most of this semester learning how to solve and analyze such systems. We will begin by understanding how to formalize them so that we can apply the techniques of **matrix theory** to solve them.

The game we've analyzed today is an example of a **Markov Process**. A Markov process has the property that

> *The future depends on the past only through the present.*

In other words, the number of pennies each player has next depends only on how many they have right now, and has no dependency on what pennies they had in the past. A Markov process is a stochastic or random process. No one can predict which pennies you will have in the next state, but if we know how the pennies are distributed, we can tell you how many you should have.