Printed Name: ____________________

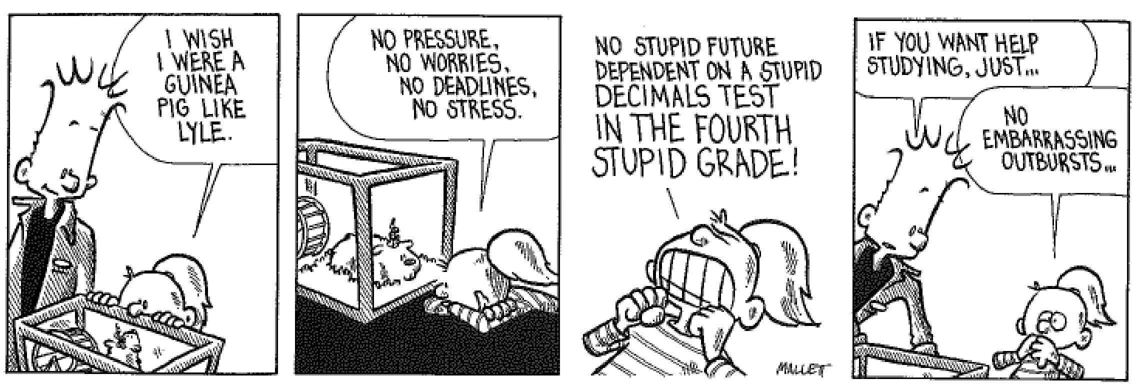
UIN: ____________ SEAT NUMBER: ____________

Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 11-15), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section and version letter of the exam on the Scantron form.**
5. Good Luck!

THE AGGIE CODE OF HONOR
"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ____________________



http://gocomics.com/frazz/

1. Compute $\displaystyle\int_1^2 \ln(x)\,dx$.

   (a) $-\dfrac{1}{2}$

   (b) $\dfrac{\ln(2)}{2} - 1$

   (c) $2\ln(2) - 3$

   (d) $\dfrac{\ln 2}{2} - \dfrac{3}{2}$

   (e) $2\ln(2) - 1$

2. Set up an integral to find the area bounded by $y = x^2 + 2bx$ and $y = bx + 2b^2$, where $b > 0$

   (a) $\displaystyle\int_{-b}^{2b} -x^2 - bx + 2b^2\,dx$

   (b) $\displaystyle\int_{-2b}^{b} -x^2 - bx + 2b^2\,dx$

   (c) $\displaystyle\int_{-b}^{2b} x^2 + bx - 2b^2\,dx$

   (d) $\displaystyle\int_{-2b}^{b} x^2 + bx - 2b^2\,dx$

   (e) None of these

3. If the average value of a continuous function $f(x)$ on $[a, b]$ is equal to 0, and if 0 is also the minimum value of f on $[a, b]$, what can you conclude?

   (a) $a > 0$ and $b < 0$

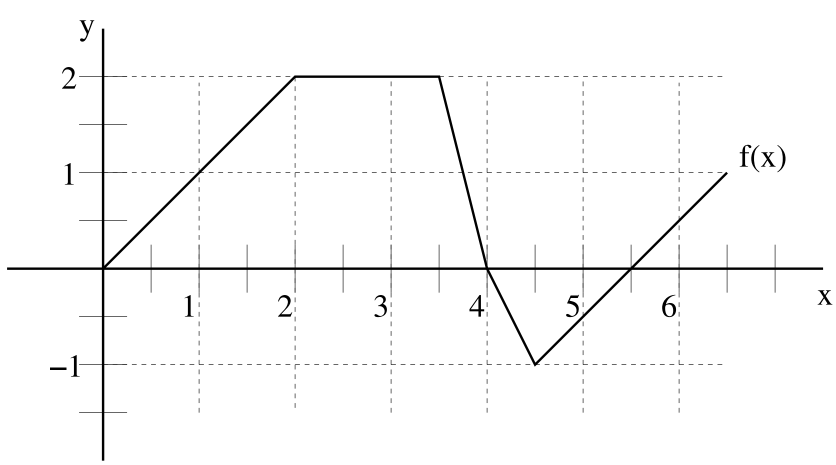
   (b) $a < 0$ and $b > 0$

   (c) $f(x) = 0$ if $a \leq x \leq b$

   (d) $f(a) < 0$ and $f(b) > 0$

   (e) None of these

4. If $g(t) = \displaystyle\int_0^t f(x)\,dx$ where $f(x)$ is pictured in the graph below, find $g(3)$ and where $g(t)$ has a local minimum.



   (a) $g(3) = 5$, local minimum at $t = 5.5$

   (b) $g(3) = 5$, local minimum at $t = 4$

   (c) $g(3) = 2$, local minimum at $t = 5.5$

   (d) $g(3) = 2$, local minimum at $t = 4$

   (e) None of these

5. Let $a > 0$. Set up an integral using **cylindrical shells** to find the volume of the solid obtained by revolving the region between the curves $f(x) = x^2$, $g(x) = ax$ around the line $x = -2$.

(a) $\pi \displaystyle\int_0^a (ax+2)^2 - (x^2+2)^2 \, dx$

(b) $2\pi \displaystyle\int_0^a x(ax - x^2) \, dx$

(c) $\pi \displaystyle\int_0^a (x+2)(ax - x^2) \, dx$

(d) $2\pi \displaystyle\int_0^a (x+2)(ax - x^2) \, dx$

(e) $\pi \displaystyle\int_0^a (x-2)(ax - x^2) \, dx$

6. A rope of length L and density ρ is dangling down from the top of a tall tower. You pull up half the rope, and when you are done, a length of $\dfrac{L}{2}$ is still dangling down. Which of the following integrals represents the work to pull this **half** of the rope up to the top of the tower? The variable y represents the amount of rope that has already been pulled up? The acceleration due to gravity is given by g.

(a) $\rho g \displaystyle\int_0^{L/2} (L - y) \, dy$

(b) $\rho g \displaystyle\int_{L/2}^{L} (L - y) \, dy$

(c) $\rho g \displaystyle\int_0^{L/2} y \, dy$

(d) $\rho g \displaystyle\int_{L/2}^{L} y(L - y) \, dy$

(e) $\rho g \displaystyle\int_0^{L/2} y^2 \, dy$

7. After an appropriate substitution, the integral $\displaystyle\int_{\frac{7}{2}}^{13} \frac{4x+1}{(2x+1)^3}\,dx$ is equivalent to which of the following?

(a) $\displaystyle\int_2^3 \left(u^{-2} - \frac{u^{-3}}{2}\right) du$

(b) $\displaystyle\int_2^3 \left(u^{-3} - \frac{u^{-2}}{2}\right) du$

(c) $\displaystyle\int_8^{27} \left(u^{-3} - \frac{u^{-2}}{2}\right) du$

(d) $\displaystyle\int_8^{27} \left(u^{-2} - \frac{u^{-3}}{2}\right) du$

(e) $\displaystyle\frac{1}{2}\int_{\frac{7}{2}}^{13} (4x+1)\,u^{-3}\,du$

8. Let $a > 0$. Set up an integral using **washers** to find the volume of the solid obtained by revolving the region between the curves $f(x) = x^2$, $g(x) = ax$ around the line $y = -2$.

(a) $\displaystyle\pi\int_0^a (ax-2)^2 - (x^2-2)^2\,dx$

(b) $\displaystyle\pi\int_0^a (ax)^2 - x^4\,dx$

(c) $\displaystyle 2\pi\int_0^a (x-2)(ax-x^2)\,dx$

(d) $\displaystyle\pi\int_0^a (ax+2)^2 - (x^2+2)^2\,dx$

(e) $\displaystyle 2\pi\int_0^a (x+2)(ax-x^2)\,dx$

9. Find the value(s) of $a > 0$ so that $f(x) = 3x^2 + 2x - 2$ has an average value of a on the interval $[-a, a]$.

(a) 1 and 2

(b) 2

(c) 1

(d) $\sqrt{2}$

(e) None of these

10. Find the volume of the solid with base bounded by the curves $f(x) = \sin(x)$ and $g(x) = -\sin(x)$ between $x = 0$, and $x = \pi$, where slices perpendicular to the x-axis are squares.

(a) 2π

(b) $2\pi + 2$

(c) $2\pi - 2$

(d) π

(e) $\pi - 1$

PART II WORK OUT

<u>Directions</u>: Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the explanations and work leading up to it.

11. Evaluate the following derivatives.

(a) (2 points) $\displaystyle\int_0^2 \frac{d}{dt}[f(t)]\,dt$

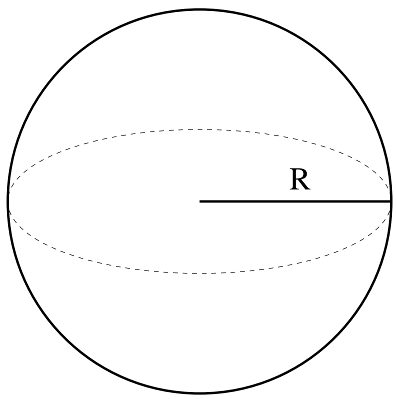
(b) (2 points) $\displaystyle\frac{d}{dx}\int_0^2 f(t)\,dt$

(c) (3 points) $\displaystyle\frac{d}{dx}\int_{g(x)}^0 f(t)\,dt$

(d) (3 points) $\displaystyle\frac{d}{dx}\left(\int_0^x f(t)\,dt\right)^3$

12. (8 points) A spherical tank with radius R is **full** of liquid with density ρ. Set up an integral for the work it would take to pump all of the liquid out of the top of the sphere. Use g for the force due to gravity.

INSTRUCTOR NOTE: Setting this up is fine, but solving this one is a lot of algebra. Leave something this hard for homework and give them a constant cross-sectional area on the exam for easier grading.



(5 points) Set up an integral to find the work to pump the liquid out if the spherical tank is **half full**.

(continued next page)

(5 points) It takes 1200J to pump the liquid out of the top of the spherical tank that is **half full**. How much energy would it take to pump the liquid out if it is full?

13. (7 points) If $f(x)$ is a continuous function, what the limit as $h \to 0$ of the average value of $f(x)$ on $[a, a+h]$? Explain how you got your answer.

14. (15 points)Find $\int \sec^3(x)\,dx$ by using integration by parts with $u = \sec(x)$. Recall that $\int \sec(x)\,dx = \log|\sec(x) + \tan(x)| + C$

Do not write below this line

Question	1-10	11	12	13	14	TOTAL
Points Awarded						
Points Possible	50	10	18	7	15	100