LAST NAME:_____ FIRST:_____

UIN: _____ SEAT NUMBER: _____

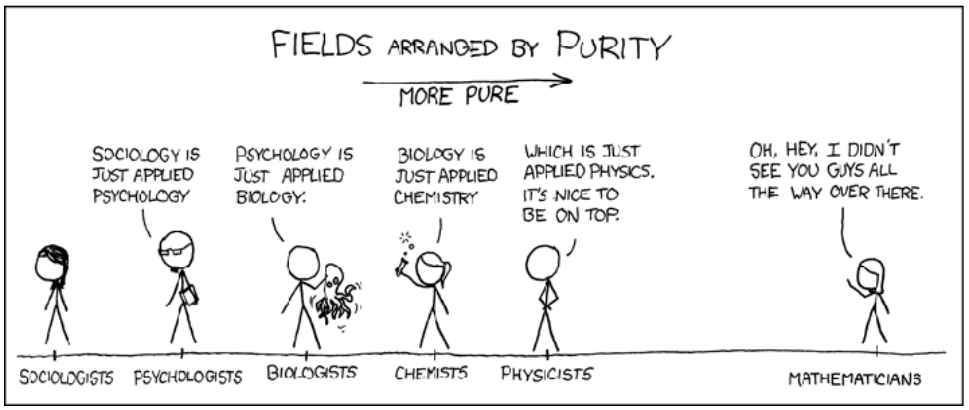
Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-13; 4 points each), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 14-19), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work and explanation leading up to it.
4. Be sure to **write your name, section and test no. on the Scantron form.**
5. Good Luck!

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: _____



http://xkcd.com

1. For which values of the constant q is the integral $\int_1^\infty x^q\,dx = C < \infty$?

   (a) $-1 < q$
   (b) $0 < q < 1$
   (c) $q < 0$
   (d) $q < -1$
   (e) $-1 < q < 0$

2. A series $\sum_{n=1}^{\infty} a_n$ has partial sum $s_n = \dfrac{2n+1}{4n^2+7}$. Which of the following statements are true?

   I. The series converges to 0.
   II. The sequence of terms $\{a_n\}$ of the series converges to 0.
   III. The Divergence Test is inconclusive for this series.

   (a) Only I and II are true.
   (b) Only III is true.
   (c) Only II and III are true.
   (d) I, II and III are true.
   (e) All three statements I, II, and III, are false.

3. Which statement is true about the sequence $a_1 = 3$, $a_{n+1} = 1 - \dfrac{1}{a_n}$?

   (a) The sequence diverges
   (b) The sequence converges to $\dfrac{1+\sqrt{5}}{2}$
   (c) The sequence converges to $-\dfrac{1}{2}$
   (d) The sequence converges to $\dfrac{2}{3}$
   (e) None of the other statements is true

4. After an appropriate substitution, and what does the integral $\displaystyle\int_2^{2\sqrt{3}+4} \frac{1}{\sqrt{x^2-8x+20}}\,dx$ become?

   (a) $\displaystyle\int_{\pi/4}^{\pi/6} \tan(\theta)\,d\theta$

   (b) $\displaystyle\int_{\pi/4}^{\pi/3} \tan(\theta)\,d\theta$

   (c) $\displaystyle\int_{-\pi/4}^{\pi/3} \sec(\theta)\,d\theta$

   (d) $\displaystyle\int_{\pi/4}^{\pi/6} \sec(\theta)\,d\theta$

   (e) $\displaystyle\int_{-\pi/4}^{\pi/3} \tan(\theta)\,d\theta$

5. Find the sum of $\displaystyle\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$

   (a) e

   (b) $e-1$

   (c) $\dfrac{e^2}{1-e}$

   (d) 0

   (e) The series diverges

6. The recursive sequence $a_1 = 2$, $a_n = \dfrac{1}{4 - a_{n-1}}$ is bounded and decreasing. Find its limit.

   (a) $2 - \sqrt{3}$
   (b) $2 + \sqrt{3}$
   (c) $2 \pm \sqrt{3}$
   (d) $\dfrac{1}{4}$
   (e) 2

7. Which of the following gives the partial fractions decomposition of $\dfrac{x^5}{x^5 - a^4 x}$?

   (a) $\dfrac{A}{x} + \dfrac{B}{x-a} + \dfrac{C}{x+a} + \dfrac{Dx+E}{x^2+a^2}$
   (b) $1 + \dfrac{A}{x-a} + \dfrac{B}{x+a} + \dfrac{Cx+D}{x^2+a^2}$
   (c) $1 + \dfrac{A}{x} + \dfrac{B}{x-a} + \dfrac{C}{x+a} + \dfrac{Dx+E}{x^2+a^2}$
   (d) $\dfrac{A}{x-a} + \dfrac{B}{x+a} + \dfrac{C}{(x+a)^2} + \dfrac{D}{(x+a)^3}$
   (e) $1 + \dfrac{A}{x-a} + \dfrac{B}{x+a} + \dfrac{C}{(x+a)^2} + \dfrac{D}{(x+a)^3}$

8. For which values of the constant q is the integral $\displaystyle\int_0^1 x^q \, dx = C < \infty$?

   (a) $q < 1$
   (b) $q < -1$
   (c) $1 < q$
   (d) $-1 < q$
   (e) $-1 < q < 0$

9. The series $\sum_{k=1}^{\infty} a_k$ has partial sum $s_n = \arctan(-2n+1)$. Which of the following statements is true?

   I. The series converges to $-\dfrac{\pi}{2}$

   II. The series diverges by the Divergence Test

   III. $\lim_{n\to\infty} a_n = \dfrac{\pi}{2}$

   (a) Only I is true

   (b) Only II is true

   (c) Only III is true

   (d) Only II and III are true

   (e) None of these statements is true

10. Evaluate $\displaystyle\int \frac{x^3+a^2}{x^2+a^2}\,dx$

   (a) $\dfrac{x^2}{2} + \dfrac{a^2}{2}\ln(x^2+a^2) + \dfrac{1}{a}\arctan(ax) + C$

   (b) $\dfrac{x^2}{2} + 2a^2\ln(x^2+a^2) + \dfrac{1}{a}\arctan(ax) + C$

   (c) $x + 2a^2\ln(x^2+a^2) + \dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right) + C$

   (d) $x + \dfrac{a^2}{2}\ln(x^2+a^2) + a\arctan(ax) + C$

   (e) $\dfrac{x^2}{2} - \dfrac{a^2}{2}\ln(x^2+a^2) + a\arctan\left(\dfrac{x}{a}\right) + C$

11. Given $a > 0$ and $b > 0$, the integral $\displaystyle\int_0^\infty \frac{ax+b}{(ax^2+2bx+c)^3}\,dx$

(a) Converges to $-\dfrac{1}{4c^2}$

(b) Converges to $\dfrac{1}{4c^2}$

(c) Converges to $-\dfrac{1}{2c^2}$

(d) Converges to $\dfrac{1}{2c^2}$

(e) Diverges

12. Which statement is true about the sequence $a_n = \sqrt{2n+2} - \sqrt{n}$?

(a) It converges to 0

(b) It converges to $\dfrac{1}{\sqrt{2}}$

(c) It converges to $\dfrac{2}{\sqrt{2}}$

(d) It converges to 2

(e) It diverges

13. What is the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{a}\right)^{-n}$ given that $a > 0$?

(a) $\dfrac{a}{2}$

(b) a

(c) $2a$

(d) $3a$

(e) The series diverges

PART II WORK OUT

Directions: Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. (7 points) Use partial sums to determine for what values of r the series

$$\sum_{n=1}^{\infty} ar^{n-1}$$

converges, and what the series converges to. You should know this answer already; the point here is to demonstrate where it comes from. Show your work and explain your reasoning.

15. Find the area of a circle of radius r.

(a) (3 points) In Cartesian (xy) coordinates, draw a diagram, write down an equation, and **set up an integral** to find the area of a circle of radius r.

(b) (5 points) Use the integral from the previous part of this problem to find the area of a circle.

16. Find the circumference of a circle of radius r.

(a) (3 points) Using your work from 15a **set up an integral** for arc-length to find the circumference of a circle of radius r.

(b) (4 points) Use the integral from 16a to find the circumference of a circle of radius r. You do not have to repeat work done to find the integral in 15b if it carries over to this problem; use words to say where you got the information you use.

17. Find the surface area of a sphere of radius r.

(a) (4 points) Draw a diagram similar to the one from 15a to show how to rotate part of a circle around an axis to get the surface of a sphere. Use the diagram and equation(s) in Cartesian (xy) coordinates to **set up an integral** for the surface area of a sphere of radius r.

(b) (4 points) Use the integral from 17a to find the surface area of sphere of radius r. You do not have to repeat work done in previous problems if it carries over to here. If so, use words to say where you got the information you use.

18. Let $a_1 = \sqrt{6},\ a_2 = \sqrt{6+\sqrt{6}}$ and $a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}$.

(a) (2 points) Assuming the sequence continues in this fashion, find a recurrence relation for a_n.

(b) (5 points) Use induction to show that a_n is bounded ($0 < a_n < 6$ for $n \geq 1$), and that a_n is monotonic.

(c) (4 points) Find $\lim_{n\to\infty} a_n$.

19. (7 points) Suppose that f and g are continuous functions which have continuous derivatives on the interval $[0, 1]$. Let C denote the curve given by the parametric equations $x = f(t)$, $y = g(t)$, $0 \leq t \leq 1$. Given that the length of C is L, compute the length of the curve given by the following set of parametric equations

$$x = g(1 - t), \quad y = f(1 - t), \quad 0 \leq t \leq 1$$

Show all your steps and explain your reasoning.

Please don't write below this line.

Question	1-13	14	15	16	17	18	19	TOTAL
Points Awarded								
Points Possible	52	7	8	7	8	11	7	100