LAST NAME:____________________ FIRST:____________________

UIN: ____________________ SEAT NUMBER: ____________________

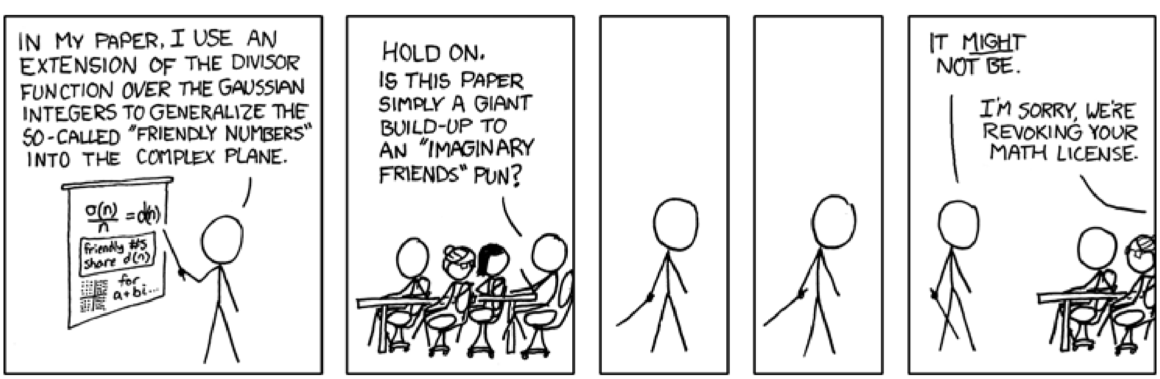
## Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-14; 3 points each), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 15-20), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work and explanation leading up to it.
4. Be sure to **write your name, section and test no. on the Scantron form.**
5. Good Luck!

THE AGGIE CODE OF HONOR

**"An Aggie does not lie, cheat, or steal, or tolerate those who do."**

Signature: ________________________________




http://xkcd.com

**Assume the scalar constants $a > 0$, $b > 0$ and $c > 0$ throughout this exam.**

1. Which of the series below is convergent?

   I. $\displaystyle\sum_{n=1}^{\infty} \frac{an-b}{bn+a}$

   II. $\displaystyle\sum_{n=1}^{\infty} \frac{an-b}{n(bn+a)}$

   III. $\displaystyle\sum_{n=1}^{\infty} \frac{an-b}{n^2(bn+a)}$

   (a) I and II only
   (b) III only
   (c) I, II and III
   (d) I, II, and III are divergent
   (e) None of the other answers is correct

2. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(n+1)\ln(n)}{n^2+2}(x-4)^n$. Find $f^{(7)}(4)$, the 7th derivative of $f(x)$ at $x = 4$.

   (a) $\dfrac{8!\ln(7)}{51}$
   (b) $\dfrac{8\ln(7)}{51}$
   (c) $\dfrac{7!\ln(7)}{51}$
   (d) $\dfrac{8!\ln(8)}{66}$
   (e) $\dfrac{8\ln(8)}{66}$

3. Find the center and radius of the sphere

$$(x+a)^2 + (y-b)^2 + z^2 + 2cz - 8c^2 = 0$$

(a) Center $(-a, b, c)$, radius $= 9c^2$
(b) Center $(a, -b, c)$, radius $= 3c$
(c) Center $(a, -b, -c)$, radius $= 9c^2$
(d) Center $(-a, b, c)$, radius $= 9c^2$
(e) Center $(-a, b, -c)$, radius $= 3c$

4. The series $\sum a_n$ definitely diverges if

(a) $\lim_{n\to\infty} |a_n| = 0$
(b) $\lim_{n\to\infty} |a_n| < 1$
(c) $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
(d) $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 0$
(e) $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$

5. The radius of convergence for the power series $\sum_{n=0}^{\infty} c_n(x+b)^n$ is $R$.

The right endpoint is included in the interval of convergence. The left endpoint is not included in the interval of convergence. What is the interval of convergence?

(a) $(R-b, R+b]$
(b) $(-b-R, -b+R]$
(c) $(-R-b, -R+b]$
(d) $(b-R, b+R]$
(e) None of these

6. The series $\sum(-1)^n a_n$ with $a_n > 0$ is definitely conditionally convergent but **not** absolutely convergent if

   (a) $\lim_{n\to\infty} a_n = 0$

   (b) $\{a_n\}$ is decreasing, $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$

   (c) $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} n^2 a_n = 5$

   (d) $\{a_n\}$ is decreasing, $\lim_{n\to\infty} a_n = 1$

   (e) $\{a_n\}$ is decreasing, $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} (n+1)a_n = 3$

7. What is the radius and interval of convergence for $\sum_{n=1}^{\infty} anb^n x^{n-1}$?

   (a) Radius $= b$, interval $= (-b, b)$

   (b) Radius $= b$, interval $= [-b, b)$

   (c) Radius $= \frac{1}{b}$, interval $= \left[-\frac{1}{b}, \frac{1}{b}\right)$

   (d) Radius $= \frac{1}{b}$, interval $= \left(-\frac{1}{b}, \frac{1}{b}\right)$

   (e) None of these

8. For which value(s) of $x$ are the vectors $\langle -x, -2, x\rangle$ and $\langle -1, 1, x\rangle$ orthogonal?

   (a) 1 and $-2$

   (b) $-1$ and 2

   (c) $-1$

   (d) 1

   (e) 2

9. Which statement is true about the convergence or divergence of $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin^2(n)}{n^2}$?

   (a) The series diverges by the Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (b) The series converges by the Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$

   (c) The series diverges by the Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (d) The series converges by the Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

   (e) None of the other statements is true.

10. Which function is equivalent to $\displaystyle\sum_{n=1}^{\infty} anb^n x^{n-1}$ inside its interval of convergence?

   (a) $\dfrac{-ab}{(1-bx)^2}$

   (b) $\dfrac{ab}{(1-bx)^2}$

   (c) $\dfrac{-a}{1-bx}$

   (d) $\dfrac{a}{1-bx}$

   (e) None of these

11. What is the angle between the vectors $\langle 1, 1, 0\rangle$ and $\langle 1, 1, \sqrt{6}\rangle$?

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$
(e) None of these.

12. How many terms of the alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$ do I have to sum in order to have the sum correct to 2 decimal places?

(a) 9 terms
(b) 8 terms
(c) 7 terms
(d) 6 terms
(e) 5 terms

13. Which series is a power series for $f(x) = \arctan(ax^2)$?

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n a^{2n} x^{2n}$
(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n a^n x^{4n+1}}{4n+1}$
(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n a^{2n} x^{4n}$
(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n a^{2n+1} x^{4n+2}}{2n+1}$
(e) None of these

14. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be nonzero vectors. Given

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|$$

and

$$0 = \mathbf{b} \cdot \mathbf{c}$$

Which of the following statements is true?

(a) $\mathbf{a}$ and $\mathbf{b}$ are orthogonal; $\mathbf{b}$ and $\mathbf{c}$ are parallel

(b) $\mathbf{a}$ and $\mathbf{b}$ are parallel; $\mathbf{b}$ and $\mathbf{c}$ are neither orthogonal nor parallel

(c) $\mathbf{a}$ and $\mathbf{c}$ are parallel; $\mathbf{b}$ and $\mathbf{c}$ are orthogonal

(d) $\mathbf{a}$ and $\mathbf{c}$ are neither orthogonal nor parallel; $\mathbf{b}$ and $\mathbf{c}$ are orthogonal.

(e) $\mathbf{a}$ and $\mathbf{b}$ are parallel; $\mathbf{a}$ and $\mathbf{c}$ are orthogonal

## PART II WORK OUT

**Directions:** Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. The power series $f(x) = \sum_{n=0}^{\infty} c_n x^n$ has radius of convergence $R$

(a) (4 points) Find formulas for the integral and derivative of $f(x)$ in terms of its power series.

(b) (6 points) Show that **either** the integral or the derivative of the power series for $f(x)$ also has radius of convergence $R$.

16. (5 points) Does the series $\sum_{n=1}^{\infty}(n+1)^{-2/3}$ converge or diverge? Clearly explain your reasoning.

(5 points) For the series above, Python estimates the partial sum

$$s_{10} = \sum_{n=1}^{10}(n+1)^{-2/3} \approx 3.324429051456158$$

Estimate the error in using $s_{10}$ to approximate the sum $\sum_{n=1}^{\infty}(n+1)^{-2/3}$.

17. (1 point) Write down Taylor's formula for the power series of $f(x)$ centered at $x = a$.

(8 points) Find the 3rd degree Taylor polynomial $T_3(x)$ for $f(x) = x^3 - x^2 + 2x - 1$ centered at $x = 1$. Simplify, but leave your answer in terms of powers of $x - 1$.

(continued)

(1 point) Complete this statement of Taylor's inequality: if $|f^{(n+1)}(x)| \leq M$ for all $x$ in a given interval containing the center of the Taylor series then the remainder $R_n(x)$ for using $T_n(x)$ to approximate $f(x)$ on that interval satisfies the inequality

(6 points) Use Taylor's inequality to estimate the maximum error in using $T_3(x)$ you found above to estimate $f(x) = x^3 - x^2 + 2x - 1$. Explain the meaning of your result.

18. (10 points) Find the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (ax-a^2)^{2n}}{a^n\sqrt{n+4}}$, where $a > 0$. Don't forget to check the endpoints for convergence!

19. (6 points) Consider the points $P$ such that the distance from $P$ to $A\left(1, \frac{5}{2}, 1\right)$ is twice the distance from $P$ to $B(1, -2, 1)$. Show that the set of all such points is a sphere, and find its center and radius.

*** As you set up the problem, be alert for features that simplify the computation. In particular, you do not have to multiply everything out.

20. (6 points) Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+2)^{3/4}}$ is conditionally convergent, absolutely convergent, or divergent. Thoroughly explain your reasoning.

Please do not write below this line.

| Question | 1-14 | 15 | 16 | 17 | 18 | 19 | 20 | TOTAL |
|---|---|---|---|---|---|---|---|---|
| Points Awarded | | | | | | | | |
| Points Possible | 42 | 10 | 10 | 16 | 10 | 6 | 6 | 100 |