Math 166 Exam 3, Form A Fall 2008

"An Aggie does not lie, cheat, or steal or tolerate those who do"

On my honor as an Aggie, I have neither given nor received unauthorized aid on this exam.

Printed name:______________________________

Signature:___________________________________

Today my seat is: Row:_______ Seat number:_______

Usually my seat is: Row:_______ Seat number:_______

- You may use your calculators, but they must be cleared of all programs before beginning this exam.
- You may not use your book or notes on this exam.
- You may not collaborate with your neighbors on this exam.
- There is no partial credit on the multiple choice or true/false questions.
- You must show all appropriate work to receive credit (especially partial credit) on the work-out problems.
- The instructor will provide additional scratch paper if needed.
- Read each question carefully.
- Write your answers to 3 significant figures if appropriate.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

$$\left[\begin{array}{c:c} I_{a\times a} & A_{a\times b} \\ \hdashline 0_{b\times a} & B_{b\times b} \end{array}\right] \longrightarrow L = \left[\begin{array}{c:c} I_{a\times a} & A_{a\times b}(I-B)^{-1}_{b\times b} \\ \hdashline 0_{b\times a} & 0_{b\times b} \end{array}\right]$$

Problems 1-6 are worth 5 points each. Mark your answers on your exam as well as on your scantron.

1. What is R_3 (row 3) of M_1 after the following row operation?

$$\begin{bmatrix} 1 & 4 & 3 & -1 \\ 4 & -2 & -1 & 2 \\ -3 & 2 & 4 & 7 \end{bmatrix} \overset{-4R_1+R_2\to R_2}{\longrightarrow} \qquad M_1$$

a) $[1 \quad 4 \quad 3 \quad -1]$ b) $[4 \quad -2 \quad 3 \quad -1]$ c) $[-3 \quad 2 \quad 4 \quad 7]$
d) $[0 \quad -18 \quad -13 \quad -2]$
e) This cannot be determined from the information given.

2. Find y.

$$\begin{bmatrix} 4 & x \\ 0 & y \\ z & 2 \end{bmatrix} - 2\begin{bmatrix} w & 0 & 5 \\ 3 & 6 & v \end{bmatrix}^T = \begin{bmatrix} 6 & 5 \\ 0 & 3 \\ 5 & 6 \end{bmatrix}$$

a) $y = 0$ b) $y = 12$ c) $y = 9$ d) $y = 15$
e) These matrices are the wrong size for addition; this cannot be determined.

3. Using the matrix equation from problem 2, find z.

a) $z = 2v + 5$ b) $z = 15$ c) $z = 0$ d) $z = 2v - 6$
e) These matrices are the wrong size for addition; this cannot be determined.

4. If these two matrices can be multiplied, identify the size of the resulting matrix and the $(2,1)$ element in their product. If they can't be multiplied, why not?

$$\begin{bmatrix} 0 & y \\ x & 4 \\ x+2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 & 5 \\ w & 6 & v \end{bmatrix}$$

a) 2×2; $3x + 4w$ b) 3×3; $3x + 4w$ c) 2×2; $6y$ d) 3×3; $6y$
e) They can't be multiplied. There are 3 rows in the first matrix, but only two rows in the second.

5. At Slow Start U, no one ever hands in the first homework assignment of their freshman year. However, if a student completes a homework assignment, there is a 90% chance that the next assignment will also be completed. If a student skips a homework assignment, there is a 60% chance that the next assignment will also be skipped. What is the percent of students who hand in the third homework assignment of their freshman year?

a) 60% b) 30% c) 40% d) 80% e) none of these

6. Given the following matrices, determine which of the following statements are true.

$$\begin{bmatrix} 1 & 0.3 & 0.2 \\ 0 & .4 & 0.3 \\ 0 & 0.3 & 0.5 \end{bmatrix} = A \qquad \begin{bmatrix} 0.1 & 0.2 & 0.6 \\ 0.4 & 0.5 & 0.3 \\ 0.5 & 0.3 & 0.1 \end{bmatrix} = B \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = C$$

I. A is a regular stochastic matrix

II. B is a regular stochastic matrix

III. C is a regular stochastic matrix

IV. A is an absorbing stochastic matrix

V. B is an absorbing stochastic matrix

VI. C is an absorbing stochastic matrix

Which of the above statements are true?

a) I and II b) III and IV c) II and IV d) II, IV, and VI e) IV and VI

On the back of your scantron mark A for TRUE and B for FALSE. Mark your answers on your test as well as on the back of your scantron.

The True/False questions are worth 2 points each.

51. a) TRUE b) FALSE All identity matrices are square.

52. a) TRUE b) FALSE If you perform Gauss elimination on a system of equations and get a contradiction, then you have infinitely many solutions

53. a) TRUE b) FALSE If a Markov process is **not** regular, it may not have a steady state.

54. a) TRUE b) FALSE Absorbing Markov processes have a steady state.

55. a) TRUE b) FALSE If T is a stochastic matrix (transition matrix), it must be square and it may have negative entries so long as the columns sum to one.

56. a) TRUE b) FALSE For any matrix M you can find M^5.

57. a) TRUE b) FALSE If you perform Gauss elimination on a system of 3 equations in 3 unknowns and you get a tautology, then you have infinitely many solutions.

Show your work on the following problems. Write your answers to 3 significant figures.

1. (8 points) Below, the first table contains information on the gallons of gasoline burned per mile (gpm) under city and highway conditions for three automobiles, a Prius, a Hummer and an Accord. The second table contains information on the miles three consumers usually travel in the city and on the highway during a typical week. Gallons per mile is just the reciprocal of the usual miles per gallon numbers reported for automobiles.

   Put these two tables into matrices, multiply the matrices, and interpret your result. You may wish to label the rows and columns of the result, and/or choose an entry in the result and describe what it represents.

	City gpm	Highway gpm
Prius	0.021	0.022
Yukon	0.083	0.052
Taurus	0.059	0.042

	Consumer 1	Consumer 2	Consumer 3
City miles	90	75	104
Highway miles	170	150	210

2. Given the following system of equations:

$$\begin{array}{rcrcrcr} x & & & + & z & = & 1 \\ 2x & + & y & + & z & = & 2 \\ & & y & - & 2z & = & 3 \end{array}$$

a. (1 point) Transfer these equations as is to an augumented matrix.

b. (6 points) Perform exactly **two** row operations on the augmented matrix you wrote down above, first obtaining a zero the $(2, 1)$ position, and then obtaining zeros in the $(3, 1)$ and $(3, 2)$ positions. Do one row operation at a time, and show the intermediate result.

3. A bakery makes three kinds of specialty cakes. The Bride's Cake (B) uses 7 cups of butter and 4 cups of sugar. The Groom's Cake (G) uses 3 cups of butter and 2 cups of sugar. The Anniversary Cake (A) uses 5 cups of butter and 3 cups of sugar. The bakery has 50 cups of butter and 30 cups of sugar and needs to decide which of these cakes to bake today.

   a. (4 points) Write this problem as a system of equations.

   b. (8 points) Find the solution to these equations as a set of parametric equations with restrictions that make sense for this problem. Think carefully about what restrictions are required to make sense for this problem.

   d. (2 points) The bakery has the same problem as above, but now wants to make twice as many Bride's cakes as Anniversary cakes. What is the new system of equations?

4. Janice only ever orders vanilla or strawberry ice cream. If she orders vanilla ice cream, there is a 70% probability that she will order strawberry next time. If she orders strawberry, there is a 40% chance she will order vanilla next time.

a. (5 points) Write down the transition matrix for this Markov process. Label the rows and columns with V for vanilla and S for strawberry, and indicate which state is the initial state (from) and which state is the final state (to).

b. (5 points) What system of equations would you use to find the steady-state distribution for this Markov process? Put these equations into an augmented matrix.

c. (2 points) What is the steady-state distribution for this Markov process? **Write your answer as a column matrix X_L with fractions, rather than decimals, for the entries.** The rows should correspond to the rows of the transition matrix you found in part a.

5. A master's degree program generally lasts two years. Each year, 70% of entering students make it to the second year of the program and 30% drop out. Of the second year students, 5% drop out, 10% must do additional study (remaining second year students for another year!) and 85% graduate. Students who drop out are not allowed to reenter the program. This information is summarized in the matrix below.

a. (7 points) Put labels on the rows and columns of this transition matrix, with D for drop out, G for graduate, 1 for first year and 2 for second year. Your labels should include an indication of which state a person is coming from and which state a person is going to (e.g. include a "from" and a "to" label). Identify the absorbing state(s).

$$\begin{bmatrix} 1 & 0 & 0.3 & 0.05 \\ 0 & 1 & 0 & 0.85 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.1 \end{bmatrix}$$

c. (6 points) Find the limiting **matrix** for this problem. Identify matrix A and $I - B$.

d. (2 points) What percentage of first year students eventually graduate?