MATH 308: Ordinary Differential Equations

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Homework assignment 5 – due Thursday 10/4/2012

An introduction to numerical methods for approximating solutions to ODEs.

You may work with a partner on this assignment, with each of you programming one of the two ODE solvers. You will then put both names on all of your work, and hand in your assignment together.

1. Implement the trapezoidal method for solving an ODE:

$$\begin{aligned} \tilde{y}_{n+1} &= y_n + (t_{n+1} - t_n) f(t_n, y_n) \\ y_{n+1} &= y_n + (t_{n+1} - t_n) \frac{[f(t_n, y_n) + f(t_{n+1}, \tilde{y}_{n+1})]}{2} \end{aligned}$$

   Save this as a function `trapezoid.m`.

2. Test it using the given `hw5_base.m` file (rename it to `hw5_1.m`), uncomment the parts on the trapezoidal method.

3. Implement the midpoint method for solving an ODE:

$$\begin{aligned} \tilde{y}_{n+1} &= y_n + (t_{n+1} - t_n) f(t_n, y_n) \\ y_{n+1} &= y_n + (t_{n+1} - t_n) \left[f\left(\frac{t_n + t_{n+1}}{2}, \frac{y_n + \tilde{y}_{n+1}}{2} \right) \right] \end{aligned}$$

   this is also called the Runge-Kutta method of order 2. Save this as a function `rk2.m`.

4. Test your `rk2.m` with your `hw5_1.m` file, uncommenting the parts for the Midpoint method.

5. Rename `hw5_1.m` to `hw5_2.m`. Find the largest step size for Euler, the trapezoid method, and the midpoint method so that the solution to the ODE $y' = y$ (already set up in `hw5_base.m`) looks like it is on top of the exact solution. How many times does each method call `'func'` to do that? Which one is best?

6. Now look at a different ODE. Save your test file as `hw5_3.m`. Solve the ODE $y' + 15y = 15t$ with $y(0) = 1$ and put the solution in the correct spot in the file. Now solve it on $0 \leq t \leq 1$ numerically. Create graphs **with a step size of 0.1** for all three methods. What happens with the Euler method in this case?

7. Can you explain why the results of the midpoint method and the trapezoid method are so similar? From the formulas we can see that they are not exactly the same.

8. Can you explain why the results of the midpoint method and the trapezoid method are better than the Euler method?

You will hand in a piece of paper with your answers to the questions posed in items 5, 6, 7 and 8.

Put your name in a comment on the first line of all your `.m` files. We are going to package up and upload the following to eLearning in class on Thursday 10/4/2012:

1. Your `trapezoid.m` file.
2. Your `rk2.m` file.
3. Your `hw5_2.m` with the largest step sizes implemented and `hw5_3.m` files.