MATH 308: Ordinary Differential Equations

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Homework assignment 6 – due Thursday 10/11/2012

Problem 1: Implement the Runge-Kutta order 4 method for solving an ODE:

$$\begin{aligned}
h &= t_{n+1} - t_n \\
k_1 &= hf(t_n, y_n) \\
k_2 &= hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\
k_3 &= hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\
k_4 &= hf(t_n + h, y_n + k_3) \\
y_{n+1} &= y_n + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}
\end{aligned}$$

I suggest starting either with the original Euler method code or with the `rk2` code you made for the previous homework. You will need your `rk2.m` code and the `euler.m` code for this homework as well; you made `rk2.m` in Homework 5. Save this as the function `rk4.m`, and test with the given `hw6_1.m` file by uncommenting the parts on the rk4 method. Once you have `rk4` working:

a) Adjust your step size in `hw6_1.m` until rk4 is on top of the solution. Attach the graph this makes to the homework.

b) Upload `rk4.m`, `rk2.m` and `6_1.m` to eLearning.

c) On the graph, give the biggest step size that you found that will work to make the `rk4` solution sit on top of the analytic solution to the ODE.

d) Also on the graph, give a listing of the three methods, `euler`, `rk2` and `rk4` from best to worst, be sure you are clear about which is best and which is worst!

Problem 2: Solve

a) $10y'' + 7y' - 12y = 0, \quad y(0) = 3, \quad y'(0) = \frac{1}{10}$

b) $y'' + 2y' + 2y = 0 \quad y(0) = 2, \quad y'(0) = 1$

c) $y'' + 6y' + 9y = 0, \quad y(0) = 2, \quad y'(0) = -1$

Problem 3: Solve the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = \alpha$$

Then find α so that the solution $y(t)$ approaches zero as $t \to \infty$.

Problem 4: The method of reduction of order
If $y_1(t)$ solves

$$y'' + p(t)y' + q(t)y = 0$$

then you can find a second solution by the **method of reduction of order**.

i. First set $y(t) = v(t)y_1(t)$, and substitute for y, y' and y'' in the ODE.

ii. The resulting ODE should be separable equation for $v''(t)$, separate variables, integrate and solve to get $v'(t)$.

iii. Now integrate $v'(t)$ with respect to t to get $v(t)$. The desired second solution is $v(t)y_1(t)$.

Use the method of reduction of order to find a second solution for

$$y'' - \frac{2}{t}y' - \frac{4}{t^2}y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$$

Verify by plugging it in that your second solution is indeed a solution to the original ODE.

The rest of these problems are not handed in. In order to master this material, I suggest doing the following problems.

Verify that if $y_1(t)$ and $y_2(t)$ are both solutions of

$$y'' + p(t)y' + q(t)y = 0$$

then so is $C_1y_1(t) + C_2y_2(t)$.

Section 3.1: As many as you can stand, but at minimum 1-23 odd.

Section 3.3: As many as you can stand, at minimum 1-21 odd, 29.

Section 3.4: As many as you can stand, at minimum 1-13 odd, 23-29 odd.