MATH 308: Ordinary Differential Equations

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Homework assignment 11 – due Thursday 11/29/2012

Problem 1: Evaluate the integral or find the Laplace transform

a) $\int_{-\infty}^{\infty} \delta(t - \frac{\pi}{2})(\sin(t) + \cos(t))\, dt$

b) $\int_{3}^{\infty} \delta(t - 2)(t^2 - 4t + 1)\, dt$

c) What is $\int_{-c}^{\infty} \delta(t - c) f(t)\, dt$ if $c > 0$? How about if $c < 0$?

d) $\mathcal{L}\{t^3 \delta(t - 1)\}$

e) $\mathcal{L}\{\delta(t - \pi)\tan(t)\}$

Problem 2 Solve

a) $y'' + 9y = -\delta(t - 2\pi), \quad y(0) = 1, \quad y'(0) = 0$

b) $y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 0, \quad y'(0) = 1$

c) $y'' + 2y' - 3y = \delta(t - 1) - \delta(t - 2), \quad y(0) = 2, \quad y'(0) = 2$

Problem 3: Recall that we've discussed that soldiers are told not to march in cadence while crossing a bridge. By solving the symbolic initial value problem

$$y'' + y = \sum_{k=1}^{\infty} \delta(t - 2k\pi), \quad y(0) = 0, \quad y'(0) = 0$$

explain why soldiers are so instructed. You may take

$$\mathcal{L}\left\{\sum_{k=1}^{\infty} \delta(t - 2k\pi)\right\} = \sum_{k=1}^{\infty} \mathcal{L}\left\{\delta(t - 2k\pi)\right\}$$

and likewise for the inverse transformation; do not add up the geometric series. Recall $\sin(t - 2k\pi) = \sin(t)$.

Your answer to this problem should include:

a) A few sentences explaining how the ODE above relates to the soldiers marching in cadence across the bridge.

b) Some math (using Laplace transforms) to solve the ODE.

c) A few sentences explaining how the solution to the ODE shows that this is not a very good idea.

Problem 4: Find the general solution to

$$\mathbf{x}' = A\mathbf{x}$$

where A is the matrix given below. For each problem

i. Give the general solution.

ii. If an initial condition is given, find the particular solution.

iii. Use `pplane8` to plot the phase plane and some solutions. Include the graphs with your answer.

iv. Classify the equilibrium solution $\mathbf{x} = \mathbf{0}$. Is it a node, a saddle point, a spiral point a circle? Is it stable or unstable?

You should be able to do these problems by hand or with a simple calculator. Only use MATLAB to confirm your answer and to plot the graphs.

a) $A = \begin{bmatrix} -2.5 & -0.5 \\ -0.5 & -2.5 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

b) $A = \begin{bmatrix} -2 & -1 \\ 2 & 0 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$

Problem 5: Find the general solution to

$$\mathbf{x}' = A\mathbf{x}$$

where A is the matrix given below. For each problem

i. Give the general solution.

ii. If an initial condition is given, find the particular solution.

iii. Use `pplane8` to plot the phase plane and some solutions. Include the graphs with your answer.

iv. Classify the equilibrium solution $\mathbf{x} = \mathbf{0}$. Is it a node, a saddle point, a spiral point a circle? Is it stable or unstable?

You should be able to do these problems by hand or with a simple calculator. Only use MATLAB to confirm your answer and to plot the graphs.

a) $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & 2 \\ -8 & 2 \end{bmatrix}$

c) $A = \begin{bmatrix} -3 & -6 \\ 3 & 3 \end{bmatrix}$

This is not easy material; I strongly recommend you do the following problems as well as these.

Section 6.5: 1-23 odd, 24-26, 27ab
Section 7.1: 1-11 odd
Section 7.2: 1-25 odd
Section 7.3: 16-21