Can Mathematical Models Offer Insight into Real World Problems?

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About me











Overpopulation

Slate =

The Big Questions: The Future of Science Made Possible by Statoil

Is Overpopulation Really the Problem?

Reality-checking Thomas Malthus, Stephen Hawking, and Dan Brown. **() () ()**

The New York Times

The Opinion Pages

WORLD U.S. N.Y. / REGION BUSINESS TECHNOLOGY SCIENCE

OP-ED CONTRIBUTOR Overpopulation Is Not the Problem

By ERLE C. ELLIS Published: September 13, 2013



Can mathematics help us to figure out if overpopulation is a problem?

Definition: A **mathematical model** is a description of a system using mathematical concepts and language.

Two models for population

Exponential growth. The change (increase) in population with time is proportional to the current population.

Equations:

$$\frac{dP}{dt} = rP$$

or

$$P(t) = Ae^{rt}$$



Logistic population growth

Logistic growth. The change (increase) in population is nearly proportional to the current population when the population is small, but the change decreases as the population gets close to its carrying capacity.



Which curve best describes the population of the world?



World population = exponential growth



If this is correct, we could run out of space.

If we continue with the exponential trend, we will be down to 1 $\ensuremath{m^2}$ of dry land per person in year 3620



http://www.hydroville.org/CUBICBOXACTIVITY

Reindeer on St. Matthew's Island: A cautionary tale



How does this curve-fitting work?



A fundamental truth about human population.

The future depends on the past only through the present.

Demographers predict and estimate from birth and death estimates.

Did the mathematical model give us insight on the question of overpopulation?

My Research: Phytoplankton population modeling

- "Plant- wanderer" photosyntesizing organisms in water
- Agents of primary production base of food web
- Depend on minerals: nitrogen, phosphorus, silica
- Sequester CO₂



Credit http://www.seos-project.eu/modules/oceancurrents/images/mt001a5_rd01.jpg

Harmful blooms can create a disgusting mess, kill fish, etc.



(All algae are plankton, but not plankton are algae...)

Credit http://toxics.usgs.gov/highlights/algal_toxins/

Experimental setup with a chemostat



Credit: http://biology.mcgill.ca/faculty/fussmann/ for the chemostat photo.

The chemostat model is from WikiMedia Commons.

Mathematical Model

- ▶ $P_i(t)$ population density of phytoplankton, i = 1, ..., n
- $N_j(t)$ nutrient (substrate) concentration, j = 1, ..., m

$$\frac{dP_i}{dt} = \mu_i P_i - \nu P_i$$
$$\frac{dN_j}{dt} = \nu (N_{\text{in}} - N_j) - \sum_{i=1}^n Q_i \mu_i P_i$$

Liebig's Law of the Minimum/Essential nutrient

$$\mu_i = \tilde{\mu}_i \min_j \left\{ \frac{N_j}{N_j + k_{sj}} \right\}$$





P. delicatissima and P. galaxiae

- Produce a neurotoxin that causes Amnesic Shellfish Poisoning
- ► Found in: Canada, US West Coast, Europe, Asia, New Zealand, Central and South America
- Frequently coexist (are found together) in nature



Credit: Phytoplankton Encyclopedia Project P. delicatissima

Data and model results - Nitrogen limitation



Species competition: data and model are not the same



Why? And what about coexistence?

- Allelopathy: chemical warfare by P. galaxiae
- Coexistence: differential sinking rates?

Model with allelopathy



This mathematical model and insight into the real world

- Good insight from the real world required!
- Math can tell us when we need more information.
- Interplay of insight: real world \leftrightarrow math.

Metacommunities: started with two REU students last summer

- "Patchy" ecosystem
- Interconnected: REU students helped set up computer model
- Linked by species dispersal
- Big question: connection to diversity in the environment?
- ► REU students: some initial experiments.



Photo credit: http://www.cottenielab.org/2011/02/ metacommunity-dynamics-in-prairie.html

Best class project

Mathematical modeling of a zombie outbreak!



Population modeling based on Susceptible-Infected-Recovered (SIR) models from epidemiology

Basic zombie model



$$\frac{dS}{dt} = p - bSZ - cS$$
$$\frac{dZ}{dt} = bSZ - aSZ + fR$$
$$\frac{dR}{dt} = aSZ + cS - fr$$

Credit: Munz, et. al. When zombies attack!: Mathematical modelling of an outbreak of zombie infection

This model is **bad news** for humans



Challenge to students!

Can you come up with a realistic hypotheses for zombie outbreak and a corresponding mathematical model in which humans survive?



What do you need to get started? What can I offer?

To get started on one of these projects:

- Differential calculus or willingness to learn
- Ideal: differential equations (not required)
- Wish list: computer programming (not required)

What can I offer?

- Have an idea, need math? Let's talk!
- Have an idea, need a (math) mentor? Let's talk!
- I want to learn and try new things.

Favorite great general audience science sites



Eff Yeah! Fluid Dynamics (Tumblr)



The Brain Scoop Videos (YouTube) The Brain Scoop (Tumblr)

Thanks for listening!