MATH 442: Mathematical Modeling

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Project 1: Lorenz Attractor (P1)
Writing 2 (W2)

Instructions You and a partner will work together to complete the tasks in this project.

One of you will write the model introduction and methods section. The other will write the results and conclusion. Your W2 grade will be based on your section; your W2 grade is an individual grade. Make sure I know who wrote what with your Group Work Assessment.

You will each create MATLAB code that creates the graphs required for this project. Make sure you use sensible names and good programming practices in your MATLAB code; you will be graded on this.

References on this mathematical topic are plentiful. Your job is to explain the topic in your own words. You should plan on reviewing at least 5-6 references; at least two of which must be from hard sources: books or journal articles. You should cite each reference in your bibliography. If you quote from a reference, you must use quotes and a citation. If you paraphrase closely or take a problem from a reference, you use a citation. If you get an idea or ideas from a classmate or your partner, include a citation for this.

Avoid excessive quotation and close paraphrasing. To accomplish this, as much as possible do not look directly at a reference while writing up your explanation, so that you use your own words. If you need to, make an outline of the major ideas and then refer back to your outline while you are writing.

Things to check before handing in the assignment

1. Have you done everything you were asked to do?
2. Is the math right? If not, get it fixed.
3. Is what you are trying to explain clear?
4. Do your graphs clearly show the conclusions you reach?
5. Are the spelling, grammar, and English usage correct and concise?
6. Do you think this is A, B or C level work?
7. Run the document through a spell checker!

By the end of this assignment, you and your partners should have a thorough understanding of the topics, the programming, and excellent explanations and examples to hand in – hopefully all A work!

Do not leave the LaTeX to the last minute!

The Lorenz Attractor We are going to investigate the Lorenz Attractor using MATLAB.

The Lorenz attractor is an example of a dynamical system. A dynamical system describes how position of a point in space (3-dimensional space in this example) changes, generally the future position depends only on the current position. It describes how a particle might move or how fluid might flow. The Lorenz attractor is derived from a simplified system to describe convection rolls in the atmosphere.

The Lorenz attractor is governed by a system of 3 first order differential equations, with three parameters (constants) in them. Given the values of the constants, and a starting value(initial condition), the evolution of the Lorenz attractor is deterministic. You can calculate where the particle will go at every time.

For certain values of the constants, the behavior of the Lorenz attractor is also chaotic. This means that if you start two particles from nearly identical initial conditions, after a long enough period of time, their trajectories will no longer be close together.

The chaotic behavior of the Lorenz attractor makes it a strange attractor and a fractal. A fractal has self-similarity in a technical mathematical sense on all scales. The types of trajectories you see in the strange attractor are repeated over all time scales.

Do a Google Search on the Lorenz Attractor and the butterfly effect, and go to the library to research these. There are good articles on it on Wikipedia, but look beyond Wikipedia. You definitely need more than one or two sources to do A work on this assignment. You may not understand everything in every article or book, but you can certainly figure out enough to complete this assignment.

In this assignment you are going to model the Lorenz attractor and determine whether or not you see the butterfly effect. Use initial starting values of $(1, -1, 1)$ and a timescale of $[0, 50]$. You must use at least two graphical/computational methods to show that you do (or do not) see the butterfly effect.

Assuming that you do see the butterfly effect, your next task is to see if if you can find a relationship between the onset of chaos and the difference in the initial conditions (starting values) for the model.

One partner will write the introduction for the project, the second will write up the results and conclusions. You will determine between the two of you who gets the methods; if one of you finds that your section is a bit short of the required 2 pages, then **you** get the methods.

You will work together to create the abstract. You will both need a bibliography for the report that contains at least one book or journal reference. One report should be generated by both partners, although the group work assessment at the end will be different. Each partner should clearly indicate in the report what section(s) (s)he wrote. Both partners upload their work onto TurnItIn on eLearning; TurnItIn will say the second parter plagiarized from the first; I will know that is not true.

(a) A 150-250 word abstract summarizing this assignment. Work together with your partner on creating the abstract; most students find abstracts tricky. One way to write an abstract is to write 1-2 sentences summarizing each section of your paper, making sure this flows into a coherent narrative summarizing the key ideas in your paper. Another is to write a summary sentence for each paragraph in your piece, and edit these together into a coherent narrative that summarizes your paper.

(b) An introduction. In the introduction you should:

  i. Explain what a dynamical system is, what an attractor is and what the Lorenz attractor is.

  ii. Explain why the Lorenz attractor is important and/or interesting scientifically. Where did the equations come from? What connection do they have with the real world?

  iii. Give the equations for the attractor.

  iv. Give the values of the constants that give the chaotic strange attractor. (These are in Wikipedia's article on the Lorenz attractor). **You should explain the meaning of the variables and constants in the Lorenz attractor as best you can.**

  v. Explain the butterfly effect.

  vi. Explain what you are doing in this assignment.

(c) A methods section explaining how you are going to model the Lorenz attractor and demonstrate whether or not you see the butterfly effect. Explain whatever calculations and graphs you will use to demonstrate whether or not you see the butterfly effect. Explain how you will get data on the relationship between the onset of chaos and the difference in initial conditions, and what model you found for the relationship. **You should include enough information about your methods so that someone competent with Matlab can reproduce your results. That you use a given differential equation solver is necessary information, that you used the plot command is not. The methods section does not include programming commands for Matlab . You should not include every detail of what you do.** There is a fine balance here to give enough information but not too much.

(d) A results and conclusion section in which you include

  i. A 3-d graph of the Lorenz butterfly that you made with MATLAB.

  ii. Whatever calculations required and and at least two graphs you that clearly show that you do or do not see the butterfly effect in your model of the Lorenz equations.

  iii. Whatever calculations and graphs you need to show the relationship between the difference in initial condition and the onset of chaos. Use the best fit line to the data and give the value of the slope and the intercept.

iv. A paragraph or two summarizing your results. Do you see the butterfly effect? How do you see it? What is the relationship between the change in initial condition and how long it takes for chaotic behavior to manifest? How do you see it?

v. **A concluding paragraph. What is the importance of your result to the general scientific community?**

(e) A bibliography where you cite your sources. I would expect a minimum of 4-6 sources, **at least one or two of which are not web resources.**

(f) Each partner submits his/her own Group Work Assessment with this assignment. You do not need to collaborate on these.

(g) Each partner individually submits a reflection, which can be a paragraph or a list of what are the most important things you learned doing this project. You may have learned some math, some MATLAB , some LaTeX, something about working with others, something about writing, etc.

You will hand in your report, group work assessment and reflection on what you learned in this assignment. This will be submitted as a PDF via TurnItIn on http://elearning.tamu.edu; the group work assessment and reflection should be an appendix or extra pages on your report and submitted with it to TurnItIn. **Your Matlab m-files will be submitted as a zip file on eLearning.**

W2 Grading This assignment is worth 50 points. See the outline above for what is required in the report sections. Your audience is a student who knows what a differential equation and initial condition is, but who doesn't remember much of anything else about them. Give some thought to the organization of your work: introduce ideas in a logical order. Explain in simple, friendly language. If you use jargon (technical terminology), explain it. Be consistent in what terminology you use throughout. It would be awesome if you and your partner could be consistent in what terminology you use in your report.

P2 Grading

- Abstract: 10 points
- MATLAB code: 20 points.

  Code should be well written. Names should make sense, organization should be logical. Code should be commented. Code should create the graphs in the report or reasonable imitations of them. Each individual student hands in his or her own code! Get help from your partner if needed, but do not copy. Put your name in a comment at the top of your code!

- Overall appearance of your report (or, as needed, your section of the report): 5 points
- Group work assessment: 5 points
- Reflection on what you learned: 10 points