Teaching Writing in a Mathematical Modeling Course

Jean Marie Linhart
Department of Mathematics MS 3368
Texas A&M University
College Station, TX 77843-3368, USA
jmlinhart@math.tamu.edu

Teaching Writing in a Mathematical Modeling Course

Abstract: Writing and communication are essential skills for success in the work place or in graduate school, yet writing and communication are often the last thing that instructors think about incorporating into a mathematics course. A mathematical modeling course provides a natural environment for writing assignments. This article is an analysis of the writing assigned in my mathematical modeling course. Formal project reports for the class teach students how to write scientific articles. Reflective assignments help students integrate their learning and apply it in new contexts. Where the assignments are traditional, I offer an analysis of problems and pitfalls in teaching this kind of writing. Where the assignments are unusual, they are thoroughly described. I share my experiences on what worked well and what didn't.

Keywords: mathematical modeling, writing, projects, reflection, portfolio, peer-review

1 INTRODUCTION

Writing and communication are central to most professional careers in STEM disciplines. Technical decisions must be made and justified; often there is more than one way to solve a problem and many considerations go into which choice is made. The ability to understand, analyze and justify a position on an issue is central to job success. Yet students in STEM fields often complain that they do not like writing and do not see its usefulness. Convincing students that good writing and communication matter and providing them with good writing and communication instruction is a crucial need in our educational system. This paper is primarily intended for instructors looking to incorporate writing into a mathematical modeling course, although it has relevance to any instructor trying to incorporate mathematical writing into the curriculum.

Patrick Bahls provides a general reference on writing and encouragement to include more writing in the quantitative disciplines in [1]. Several ideas for this class were inspired by this, in particular using the final portfolio rather than a final exam for a class and the idea of having students write in a two-person dialog format. *PRIMUS* has featured many articles on writing in mathematics courses; some that might be of interest are increasing student's conceptual understanding in calculus through writing [2], writing to learn [8], the whys and hows of getting started in incorporating writing in a mathematics classroom [9], and assessment of writing in mathematics [7]. Writing and communication are not only valuable in the workplace; they are also valuable, if underutilized, tools for increasing student learning.

Writing is particularly suited to a mathematical modeling class. A mathematical model, particularly in the sciences and engineering, describes a system using mathematical concepts and language. Making assumptions, justifying, explaining and criticizing the mathematics used is crucial to the process, and this, in turn, requires writing and communication. Mathematical modeling is where mathematics meets the *real world*, and it is essential that students understand and can explain how the two are connected.

In this article, I discuss the writing in my mathematical modeling course, giving an analysis of problems, pitfalls, and the successes I have had. I start by giving an overview of the course goals. I then present the structure of the writing assignments in the order in which we go through them in the class. I give details of the unusual writing assignments and their impact on the students. The first and last assignments are informal and reflective in nature. The three project reports in the middle develop students' ability to do authentic, professional disciplinary writing in the form of a scientific article. Since this type of writing is familiar, I do not give detailed assignment descriptions of the projects and reports, and I focus on my observations and experiences. The final portfolio has students highlight their key learning experiences of the semester. This identifies what they are getting out of the course. For details of specific assignments and requirements, see [4]. Next to last, I discuss how I handle peer-review. Finally, I examine how the writing in the course serves the students who take it.

2 COURSE OVERVIEW

The primary goal for the course is to introduce students to mathematical modeling, starting with understanding the modeling process. Students work through several example mathematical models, and learn how to get modeling results from a computer algebra system. I want students to learn how to use critical thinking in a modeling situation, which often means questioning and refining modeling assumptions and evaluating the modeling results. Due to this emphasis on critical thinking, it was a logical step in my mind to make this course writing intensive.

My secondary course goal is to develop skills for success in professional situations or graduate school. Professional work, research work and writing are often collaborative and peer-reviewed. Students receive little formal instruction in collaboration, working effectively in groups, and peer-review. As I explain to my students when we discuss the first day handout, our topic is mathematical modeling, but within this context I want to provide them with knowledge, skills and experiences that they will find useful in their careers.

This class is taken primarily by seniors. Usually half or more of the class is engineering majors taking it to complete a mathematics minor. Required prerequisite course work is differential equations and linear algebra, both of which require completion of the three semester calculus sequence.

A mathematical modeling course could cover many more models and aspects of modeling. In the choice between covering a lot of material and covering it deeply, my decision was to go deep into two areas and allow students some choice into a third area for the final project. The coverage issue is discussed in [10].

3 WHAT IS MATHEMATICAL MODELING?

I always start the semester by giving the students a detailed outline of the mathematical modeling process, one containing about six main points each of which have two or three subpoints.

I expected this overview of mathematical modeling to be an easy learning outcome; this is a straightforward process with no math involved. The first semester I taught the class, I assigned the students to write an essay in response to this as the first deliverable in the final portfolio assignment. Only one student went back to the notes from the beginning of the class to discuss items from the outline. Most of the answers were along the lines of "Mathematical modeling is programming in MATLAB," reflecting what the students spent their time struggling with, but not what I wanted them to know.

The next time I taught the class, during the first two weeks of the course, I assigned a one page essay describing mathematical modeling and requiring students to use exactly two example mathematical models in their essays. The responses to this assignment were better, but I still observed two problems. First, students struggle to understand quantification, *i.e.*, the art and science of making something not naturally measured in numbers numeric, and often used quantification's multiple meanings in multiple contexts. Second, students tend to tack on the two example mathematical models at the end, rather than using them to help explain the process of mathematical modeling. Quantification, in particular, is best explained with an example. A third issue is that reading many similar essays on the process of mathematical modeling rapidly became tiresome.

I received wise advice to assign writing that I am interested in reading. I applied that advice to this assignment. I changed it again, this time asking students to imagine that a friend, a parent, or a person sitting next to them on the bus asks them what mathematical modeling is, and to write a two-person dialog of the conversation that ensues. One character explains what mathematical modeling is and the other character asks questions and makes comments. I told the students that they were encouraged to be creative. This is now a writing-to-learn assignment rather than an expository essay written by an expert.

The results of this writing prompt were insightful and incredibly varied. Students tried to connect and integrate the process of mathematical modeling to their real lives, every day problems, research experiences, or internships. I saw the process of the transfer of their knowledge, and many signs that deep learning was occurring.

The one modification I need to make to this writing prompt is to allow students to use only one example mathematical model, since several students are now giving excellent descriptions of mathematical modeling while describing one example model in detail, interwoven with the more abstract explanation of what mathematical modeling is in general.

In retrospect, I can clearly see why assigning this essay as a dialog worked so much better than my first two ideas. As instructors we tend to regard the big picture of what we are doing, high-level thinking, as being *easy* because there are no complicated calculations involved with it. To a student, who has minimal exposure to the topic, seeing the forest from the trees is anything but easy. In writing the dialog, students learn what mathematical modeling is. In a dialog students can ask and attempt to answer their own questions about the muddy pieces. Further, when students personalize their work, as they are forced to do here, they do more internalization and thinking about what the material means to them, which is exactly what I want to have happen.

This assignment sets the stage for the rest of the course. I want students to reflect on the various parts of this process as they see them in the modeling projects.

4 PROJECTS

The main meat of the course is working through and writing up three projects on mathematical modeling. Students see the assumptions that underpin the mathematical models and explore some aspect of mathematical modeling in completing each project. Projects provide an introduction to research and writing a research article. They are done in groups, just as research done in universities and projects done in companies are generally collaborative. However a single student is primarily responsible for a section of the report on the project. There are some areas of shared responsibility, which again mimics the way people collaborate in the real world.

The specific projects chosen are less important than the scaffolding of the understanding of mathematical modeling and report writing. My first project has students work on identifying the butterfly effect and exploring chaos in the Lorenz Attractor, which is a greatly simplified model for atmospheric convection. This project has many specific directions regarding the written report. Students are given an explicit list of information to put in the introduction, methods, results, and conclusions sections.

The second project has students explore three models (exponential, logistic and Gompertz) for population growth with human population data; it has some specific directions, as with the first project, but fewer of them. It also introduces a change in the standard report format. We compare results for the three models fit to three datasets. Everyone uses the United States population data, and students choose two other population datasets; such as a city, state, province or country. I suggest a format where the report has three results and conclusions sections, one for each dataset analyzed, then a general conclusions section where the group discusses how the three mathematical models perform overall.

For the third and final project the students have a choice of what to work on, either a project of their own devising or one of several options that I have designed. Popular choices include applying susceptible-infected-recovered (SIR) disease models to a zombie outbreak or exploring the efficient portfolio frontier for investing money in the stock market. There are few directions given for the third project report; students are now expected to figure much of this out from the work they did previously.

Along with each project, students are required to do some reflective writing, which helps students to see and understand the ways that they have grown over the course of the project or the semester. "The ability to apply learning to new contexts is developed when students have opportunities: to meta-reflect (to think about how and what they have learned and the process of learning itself), to integrate their learning (to think about how one learning experience connects to another), and to consciously connect learning to life experience," ([6], pp. 2-3)

4.1 Project reports

We think students are all familiar with the standard research paper format, which includes an introduction, methods section, and a results/conclusions section, since they have seen this in the form of standard laboratory reports in their lab science courses. However, the format of a report is generally not understood explicitly by students, only implicitly, and often that understanding is unclear. Students will need guidance to understand the parts of a paper and the organizational choices that govern what goes into each section. What information to include and where you put it are central questions that students have. What to do and why it is done that way are key points where the instructor must provide mentoring.

4.1.1 Introduction

The introduction of a research paper should interest the audience, and give the audience all relevant background information about the problem addressed in the paper, and state clearly what problem is addressed. This is a difficult section for students to write. Challenges students face when writing an introduction section include

i. Identifying their audience and reasonable expectations of the audience,

ii. Piquing the interest of the audience,

iii. Learning new material,

iv. Identifying the equations and background mathematics they are using, and

v. Figuring out how to organize the material into a logical structure, so that they don't use a concept before explaining what it is.

Students struggle with the introduction because often this is the first time they have been asked to explain where mathematics comes from and what it means, or to use mathematics to explain an idea. Often in their other writing, they have assumed they are writing for their teacher (an expert reader), rather than a peer, and so they aren't used to taking into account what their reader would and would not know. Add to that the fact that they are writing about material that they have only just learned, and it is easy to see why this section is difficult.

Peer-review (see below) is very valuable for improving introduction sections. Since the audience is someone who has not taken our class, if a peer in the class is confused by a piece of writing, the writing needs to be improved.

4.1.2 Methods

Methods sections are challenging, because they are so different from what the students would produce in a laboratory report for a science class, where the methods are long and detailed and the student has a description that he or she is supposed to reference in writing up his or her report. In a mathematics class, we teach them what to do, but there is no list for them to refer to in writing their papers. Additionally, we teach students many programming steps that are specific to whatever computer algebra system we are using. These steps are needed for the computer algebra system, but they are not the essence of the solution to their modeling problem. Thus they should be omitted from the methods discussion. Students have a hard time differentiating between material that is essential for them to know and material that it is essential to convey to the audience. It is very hard for students to understand what to include and what not to include.

I tell my students that methods sections tend to be very brief; they should give enough information for an intelligent person to reproduce the results in the paper, but without providing step-by-step programming advice. Include names of the mathematical methods we employed, and only include names of specific routines in the computer algebra system when there are multiple options that can influence results. For example, computer algebra systems often have several different differential equations solvers, and each will produce different results. Since the choice of a differential equations solver affects the replicability of results, we include the specific name. We may also have to give some mathematical guidance as to how to employ specific techniques. We may use linear least squares to get parameter estimates, but if we are not dealing with a linear equation, we have to explain how to put the equation into linear form.

Even with much repetition, these instructions are not clear to them. The most typical problem encountered in student writing of methods sections is putting in too much detail. I've found the best way to address this is to have an explicit class discussion about what belongs in the methods and what does not for each project. Students also iron out some of these issues by doing peer review.

4.1.3 Results

Results sections, in contrast, tend to be much easier for students to write. Results generated from their computer code directly correspond to what they should be writing about. Some guidance is required concerning the development of graphs and explaining the meaning of graphs, but overall results sections are much more straightforward for students to write and for instructors to teach than either the introduction or the methods section.

4.1.4 Conclusions

Conclusions sections are middle ground in difficulty between introduction and methods and the results section. Students have had the need for conclusions drilled into them by previous writing teachers, and saying whether something worked or not is straightforward. Critical thinking exercises for conclusions are more challenging; it is in a conclusions section that students critique what they've done and offer ideas for improvement. In a mathematical modeling context, this means critiquing the assumptions of the mathematical models and suggesting how to make better assumptions. Overall this exercise and the fact that it goes in the conclusions section makes sense to students. Actually performing the task is harder because there is no one right answer, and similar to the methods section, there isn't a formula for what to say.

Peer-review is very valuable here too, as students can see how others concluded and criticized the models.

4.1.5 Abstracts

My class is generally the first exposure students have to writing abstracts.

Although an abstract is the first thing in a report or scientific article, I ask students to write it last. I always assign them to write an abstract as a group. In working collaboratively, I hope to provoke discussion with project partners about what should go in the abstract and what should be left out.

The abstract is the single most difficult part of a paper for students to write, probably because it is so brief and so abstract (pardon the pun). We condense a 5-10 page paper into 100-200 words. I give them the following scaffold of items to include and questions to answer.

1. Introduction: why is this important?
2. Problem statement: what are you trying to do?
3. Methods: what are the essential methods?
4. Results: what are the key results?
5. Conclusions: what does it mean?

I have seen abstracts improve when we go over a set of sample abstracts with the scaffold in mind trying to pick the best and justify our choices.

4.2 Types of Reflective writing

Each project also requires two short pieces of reflective writing. First, since projects are collaborative, each individual writes a group work assessment to evaluate how they worked together to complete the project. This helps keep students accountable for contributing to their group, and helps them reflect on what makes groups work well or poorly. Second, I ask students to reflect on what they learned in completing the project. My first intention is for students to identify their learning experiences, what went well, what went poorly, and what they would like to do better next time. My second intention is to help students be clearly aware of how they have grown over the course of the project and semester. We all tend to minimize or forget what we learned and how we have grown as we go forward.

4.2.1 Group Work Assessment

The group work assessment's functionality is essential in maintaining accountability for individual contributions to the group project.

Each student is asked to hand in

1. A list of what the student contributed to the project,
2. A list of what the other group members contributed to the project,
3. Any particular problems or successes the group had in working together,
4. Anything the student would do differently if s/he could do it over,
5. Whether or not the student would want to work with the rest of this group again, and
6. If 100 dollars were to be distributed among the group members according to the contribution to the group project, how does the student think this should be fairly distributed, and give some short reasons why.

Different students have different perspectives on how things went, but the group work assessments are fairly consistent. This helps students to monitor their own contributions to the group and assess what works and does not work when working with others. With this information as well as my own observations, I can adjust grades to reflect the contribution to the project and/or make more appropriate partner assignments in the future. The most important outcome is that this helps students identify key components of the collaboration process and the things they need to do to be more effective in working with others in the future.

4.2.2 Reflection

The last time I taught this course, I began asking students to include a section at the end of each project discussing what are the most significant things they learned in completing the project. This part of the assignment did not go smoothly at first; students often have little experience with reflection and reflective writing.

To remedy this, I gave guidance in the form of a long list of questions they might answer in their reflection. I thought that they would pick from those questions and address a few. Many students answered every question briefly instead of addressing a few important items in depth.

I refined further by giving students a format for the reflection along with the questions. I instructed them to write a brief letter to me. I saw substantial improvement. Requiring them to write it as a letter seemed to help naturally focus on the audience (me) and what I might want to know about their learning, but this format also allowed them significant freedom to address what was most important to them.

An added bonus is that students can use these reflections to put together their final portfolios at the end of the semester.

5 FINAL PORTFOLIOS

A final portfolio for a class should highlight what the student learned over the course of the semester. My initial final portfolio assignments were far too long; I tried to capture *everything*. I asked them to write an essay describing what mathematical modeling is, to provide and discuss a writing sample, to provide and discuss a code or graph sample, and to write a letter to a future student describing how to succeed in the class. I am not sure I succeeded in much besides tiring students and myself out.

This has been shortened every semester. The latest change was inspired by the realization that what I most enjoyed reading in the portfolios were the letters to future students. While I still have the students include a writing sample and a code/graph sample, I now ask them to discuss their choices and any revisions in the writing sample in a two page letter to me which should discuss generally what they learned in the class. I continue to ask them to write a one page letter of advice to future students. While I am asking less, I am receiving more and better feedback about what they learned in the class and what about the class most influenced my students.

6 PEER REVIEW

Putting students in the teacher's chair and asking them to provide feedback to each other empowers them. Peer review can provide students with detailed formative feedback, and teach them how to look at their own writing with a critical eye. However, students don't know how to evaluate their own writing, much less their peer's. The instructor must first give some feedback on their work and general guidelines to help them in evaluating each other. Success also depends on a classroom environment where students feel safe and supported by their classmates. Finally, it is important to realize that students will not perform peer-review unless you make it a priority for them; I have them do this in class.

Before having the students review each other's work, I read through the rough drafts so that I have a general idea of what issues concern me the most. I give some specifics in telling students what to look for, how to comment, and reiterate the importance of praise. We are writing for an audience that does not know as much as we do, so it is particularly important for students to comment on anything they find confusing. I also encourage them to comment if they see something that is explained well.

Our first peer-review assignment is the two-person dialog about mathematical modeling. Students have a clear idea what points they are required to present which gives them a shared context for giving feedback. The dialogs are so different that they cannot help but get a new perspective on the topic by looking at others' efforts. This sets a good stage for future success.

For the project reports, students exchange papers with someone writing on the same topic they are, and, also with one person writing a different section of the report. I am trying to balance knowledge with a fresh perspective. Students do not exchange with their own group members during class time; giving mutual feedback is part of being a group member. They are expected to do this on their own.

Students also need instruction about ethics and peer-review. What should students do with an idea in someone else's paper that would be useful for her or his own? The first semester I had a student use another student's ideas without citation and without the other student's knowledge or permission. Now I tell my students to ask, because that is polite, and then cite the other person. I make sure they have some examples of citing a paper from a class or personal communication.

7 CONCLUSIONS

In terms of awards and publications, this class has been an outstanding success. Daniel Miller and Andy Cho, from the Fall 2011 class, had papers on their final projects published in *Explorations*, the undergraduate research journal at Texas A&M ([5], [3]). Miller also won the Writing Center's video competition for the video he made on chaos theory and the Lorenz attractor. Cho received an honorable mention in the corresponding writing competition. Cho took first prize for his poster at Student Research Week 2012 at Texas A&M for undergraduate mathematical research. Miller continued his research as part of the Undergraduate Research Scholars program, and was designated as a runner-up for outstanding undergraduate STEM thesis in Spring 2013.

Miller was selected as a Goldwater Scholar. Other students presented their course projects at national mathematics meetings, and one even won a prize for best applied mathematics presentation by a student.

Forcing students to write is a huge part of this success. Students have submitted writing to *Explorations* and the Writing Center's competitions because they already had it done for class.

Outstanding students will always produce outstanding work, so a good question to ask is how the class is serving a weak student or an average student.

The course has helped to improve writing skills of students whose first language is not English. Their overall understanding of the mathematics was strong, and their motivation to perform well in the course was also strong. These students would take the time to seek out feedback and take it into account, and often saw substantial improvements. The University Writing Center provides peer-mentor help with papers, a service that motivated students used and found helpful. The Writing Center provides help with all levels of writing, from grammar and punctuation, to organization, audience and tone.

The course is not well suited for remediating students with weak skills overall, especially if they are accompanied with a lack of motivation. The structure of the course is for longer assignments that require planning ahead. Last minute work plus weak fundamental skills are a deadly combination that leaves no time to get help from a peer, the professor, or the University Writing Center.

That said, I have had such a student fail herself in the course, and turn around and retake it the next semester with a much improved work ethic. While she struggled with writing and mathematical skills, she got through the course successfully the second time. She wrote in her final portfolio, "I am proud of the work and effort I put into this class ... I feel that I have made you proud of me and the improvements that I made. I put my heart into everything I did ... and proved to myself that the class is not as hard as I made it out to be."[1]

The course seems to serve average students very well. Early course

[1] The student gave permission to use her story and quote in this article.

feedback always contains a lot of complaints about how the class is too hard and too much work, and yet in the final portfolios and final course evaluations, students expressed their thanks and enjoyment for the class. In the portfolios, my students told me about their wonder at chaos theory in the Lorenz attractor and the connection to predicting the weather. They told me about being disappointed at first to get no clear *superior* model in the population project, until they realized that given the complexity of human populations in comparison to the simplicity of the models that there was no way these models should be able to give superior results. Many students said they enjoyed peer-review; they got good ideas from looking at how others structured their work, and good criticisms and insights on their own work. They learned how to put a *real* scientific article together for the first time, had to work harder to explain things than ever before.

One student wrote,

> Writing in this class has been a new twist from writing in English classes. As you have probably seen in my writing, it is not the best. All of my life I have disliked writing, because it has always taken me very long periods of time. ... On the other hand, I have always liked math, and it has always been my strong point. Therefore writing about math has motivated me and encouraged me to write better in less time.[2]

What I found particularly telling in the final portfolios is that students told me how they applied the ideas they discussed in their dialogs on mathematical modeling to how they approached their final projects. Seeing this feedback and the long-lasting influence of that initial dialog tells me that the writing is helping students organize and better understand what they are learning.

[2]The student gave permission for the use of this quote.

ACKNOWLEDGMENTS

I am grateful to Sue Geller for substantial help in editing this paper. She is a generous mentor, with a keen eye toward concise and readable writing. Daniel Miller, originally a student, is now transitioning to a colleague; he provided comments on this paper and careful proofreading. Carolyn Sandoval and her Course Design Series taught through Texas A&M's Center for Teaching Effectiveness provided a structure that allowed me to think through my initial design of this course. Rebecca Clark, Carrie Eaton, and Peter Howard all gave initial feedback on this manuscript and encouraged me to keep writing and revising. Thank you all.

REFERENCES

[1] Patrick S. Bahls 2012. *Student Writing in the Quantitative Disciplines: A Guide for College Faculty.* Jossey-Bass. ISBN 978-0470952122.

[2] James Beidelman, Doug Jones and Pamela Wells 1995. Increasing Students' conceptual understanding of first semester calculus through writing. *PRIMUS.* 5(4): 297–316.

[3] Andy Cho 2012. Surviving a zombie apocalypse. *Explorations.* 4: 39–41. `http://www.scribd.com/doc/125174630/ExplorationsIV-Opt` Accessed 29 May 2013.

[4] Jean Marie Linhart 2013. Website with course materials for Math 442 (Mathematical Modeling) `http://www.math.tamu.edu/~jmlinhart/m442`. Accessed 17 Jan 2014.

[5] Daniel R. Miller 2012. Powering your water heater with solar energy. *Explorations.* 4: 42–46. `http://www.scribd.com/doc/125174630/ExplorationsIV-Opt` Accessed 29 May 2013.

[6] Melissa Peet 2012. The Integrative knowledge portfolio process. *Digication.* Linked on `https://bu.digication.com/JamesWolff/The_`

Integrative_Knowledge_Portfolio_Process. Accessed 29 May 2013.

[7] Thomas W. Rishel 1994. Assessment of writing in mathematics. *PRIMUS*. 4(1): 39–43.

[8] D. Sabrio, S. Sabrio, and G. Tintera 1993. Writing to learn and learning to write mathematics: an experiment. *PRIMUS*. 3(4): 419–429.

[9] Bethany Seto and David E. Meel 2006. Writing in mathematics: making it work. *PRIMUS*. 14(3): 204–232.

[10] Stan Yoshinobu and Matthew G. Jones 2012. The Coverage Issue. *PRIMUS*. 22(4): 303–316.

BIOGRAPHICAL SKETCH

Jean Marie Linhart completed her Ph.D. from the University of Texas at Austin, and then worked in the digital imaging and mathematical/statistical software world for 10 years. She eventually returned to teaching at Texas A&M University, where she brings her knowledge of the "real world" as well as the expectations of academia into her classroom to best prepare students for the future. The most interesting class she's taught at Texas A&M, aside from the mathematical modeling class, is a first-year seminar on geocaching. She bikes to work almost every day and intends to continue to do so far into the future.