Can Mathematical Models Offer Insight into Real World Problems?

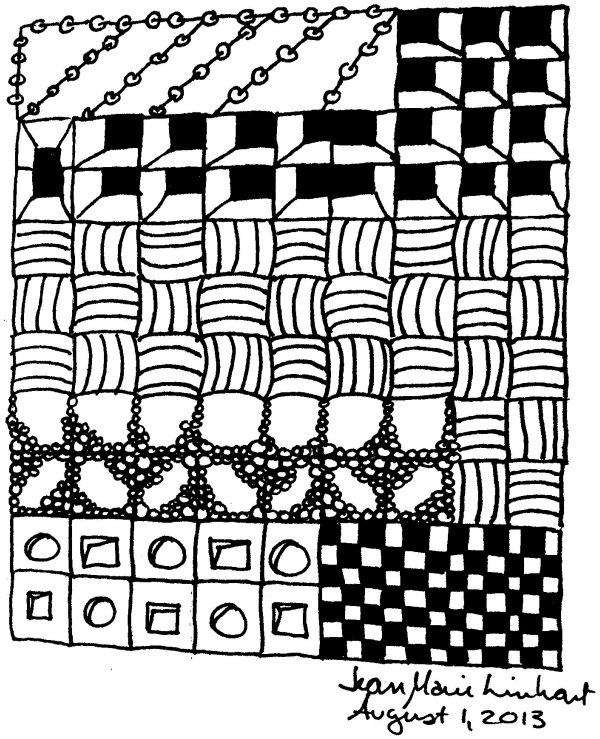
Jean Marie Linhart

February 3, 2014

Given at Central Washington University.

About me


MAA Math Fest


Jean Marie Linhart
August 1, 2013


THE NEXT 500 YEARS
Life in the Coming Millennium
The Next 200 Years
THE NEXT 100 YEARS
GEORGE FRIEDMAN
PACE THE NEXT 100 YEARS
THE NEXT FIFTY YEARS
50 years from today
MAN: THE NEXT 30 YEARS
SPACE
AMERICA'S NEXT TWENTY YEARS
UPCOMING CHANGES
Energy
The Next Twenty Years
Evangelism • The Next Ten Years • Wirt
THE NEXT FOUR YEARS


Diet
Coke

Overpopulation

Slate

The Big Questions:
The Future of Science Made Possible by **Statoil**

Is Overpopulation Really the Problem?

Reality-checking Thomas Malthus, Stephen Hawking, and Dan Brown.

401 201 90

The New York Times
The Opinion Pages

WORLD U.S. N.Y. / REGION BUSINESS TECHNOLOGY SCIENCE

OP-ED CONTRIBUTOR

Overpopulation Is Not the Problem

By ERLE C. ELLIS
Published: September 13, 2013

Can mathematics help us to figure out if overpopulation is a problem?

Definition: A **mathematical model** is a description of a system using mathematical concepts and language.

Two models for population

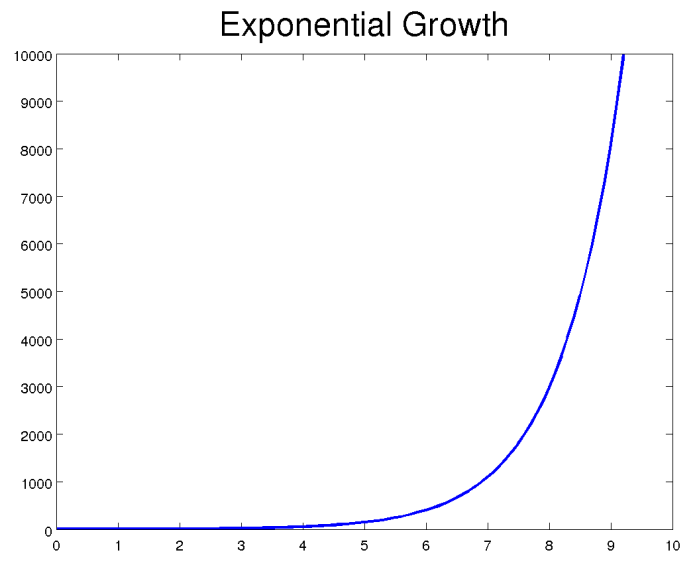
Exponential growth. The change (increase) in population with time is proportional to the current population.

Equations:

$$\frac{dP}{dt} = rP$$

or

$$P(t) = Ae^{rt}$$



Logistic population growth

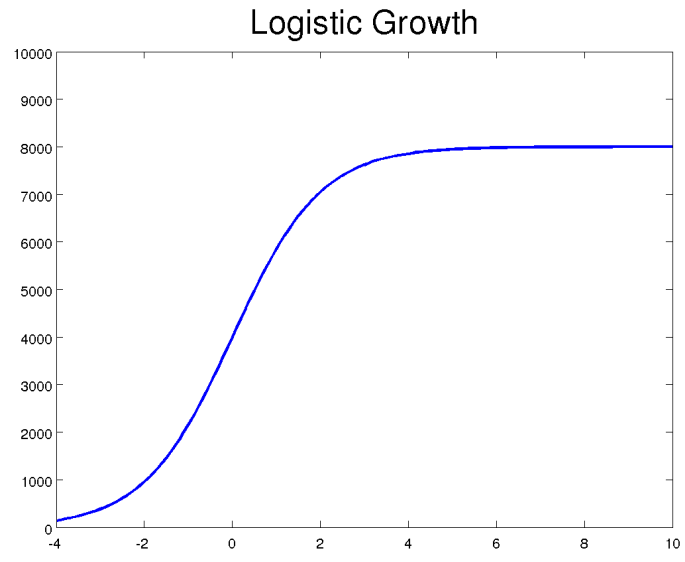
Logistic growth. The change (increase) in population is nearly proportional to the current population when the population is small, but the change decreases as the population gets close to its carrying capacity.

Equation:

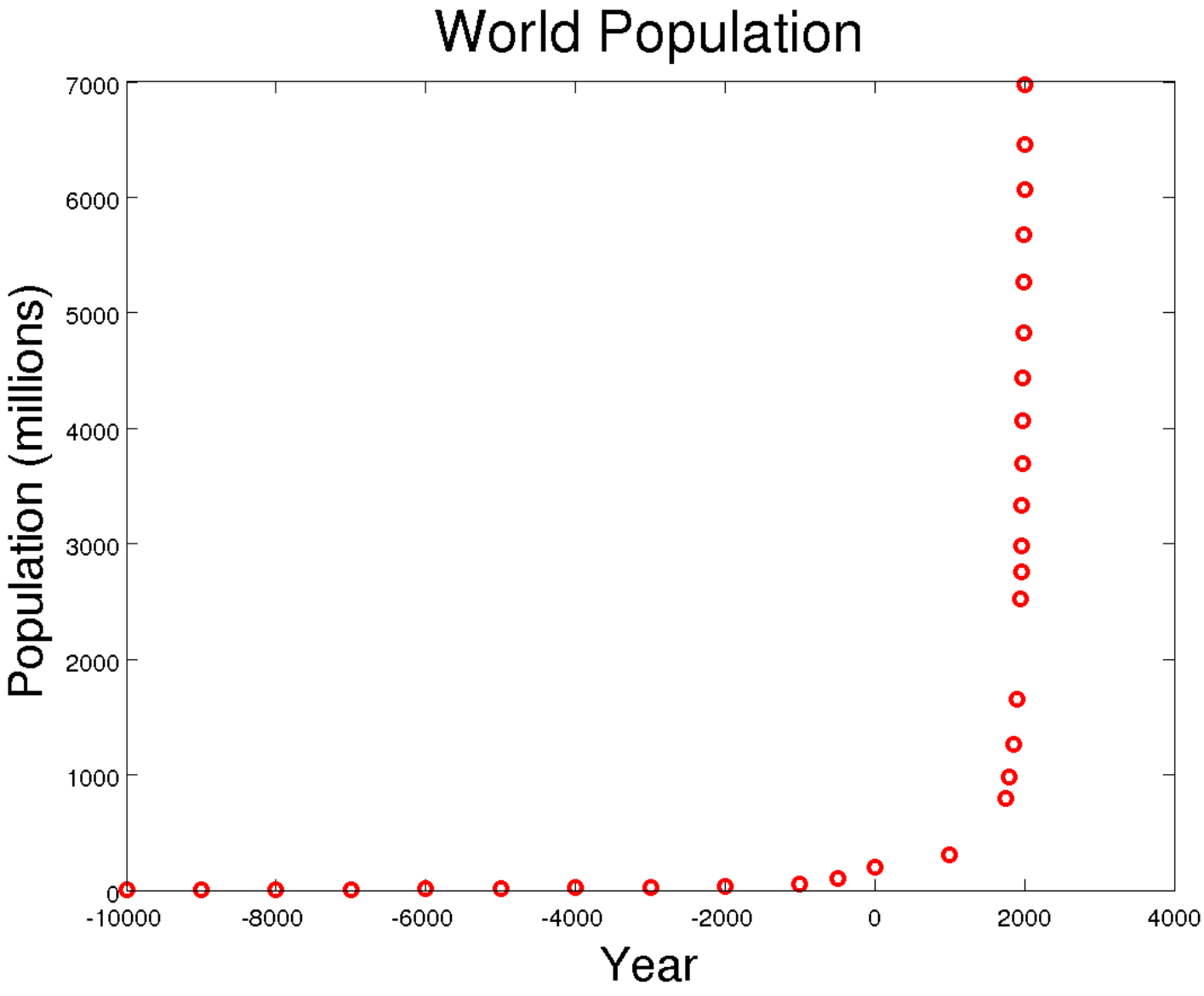
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{L}\right)$$

or

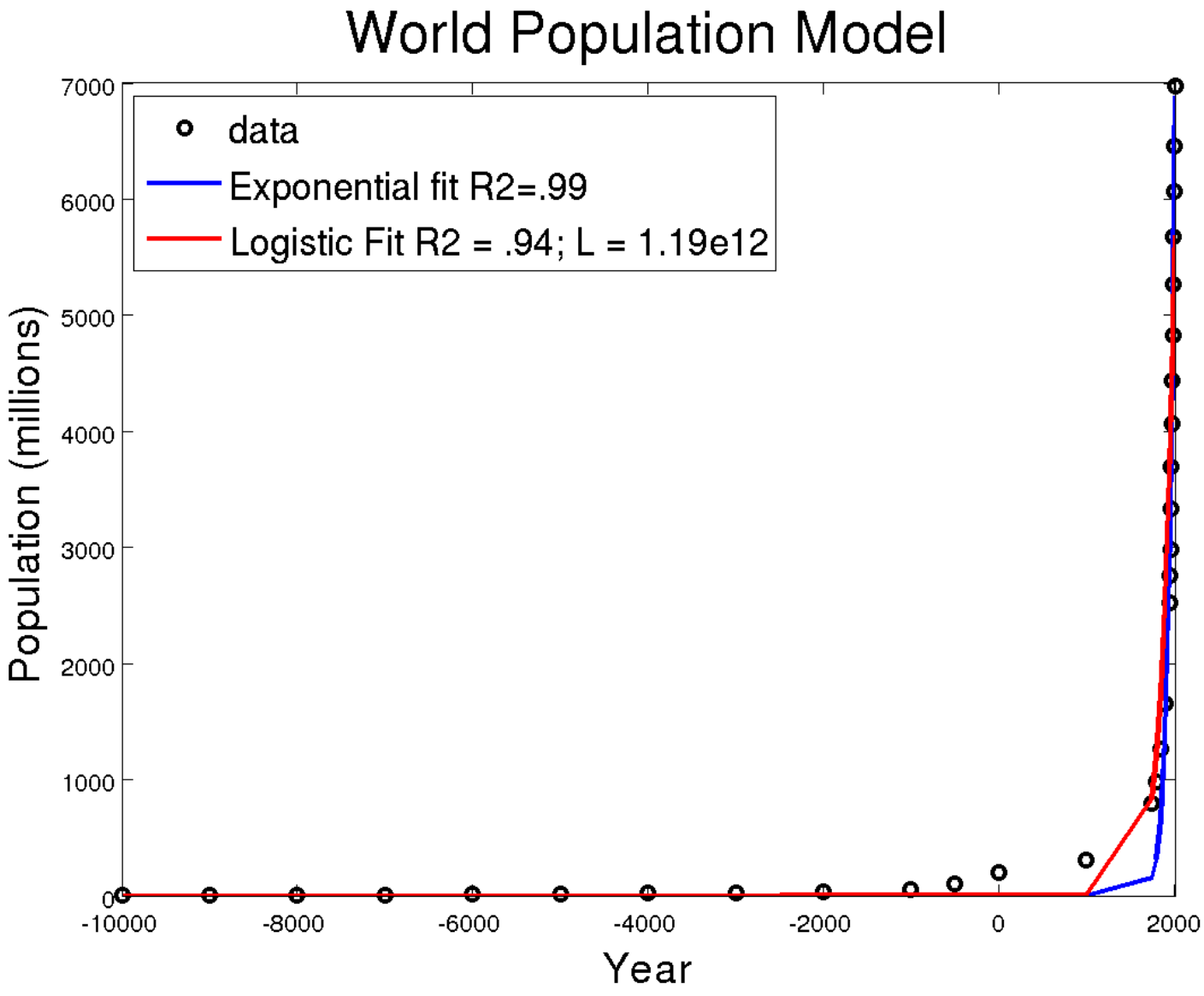
$$P(t) = \frac{L}{1 + Ae^{-rt}}$$



Which curve best describes the population of the world?

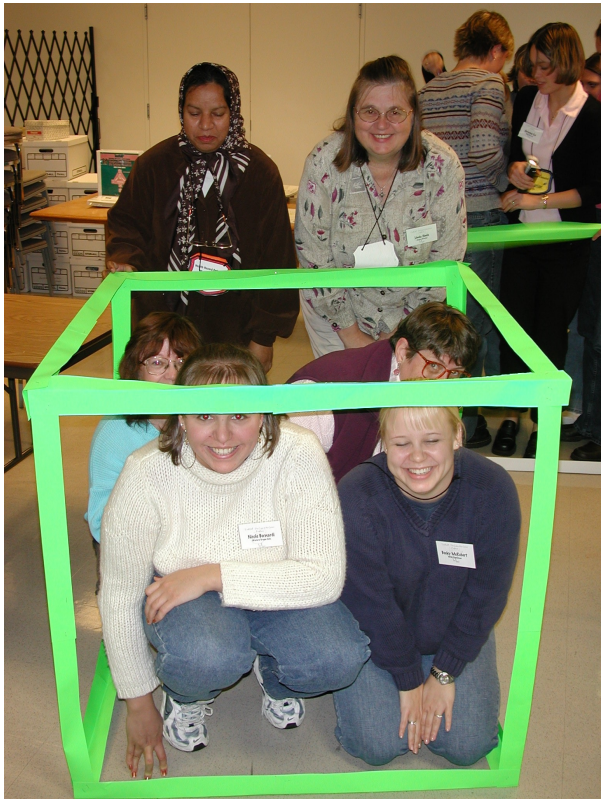


World population = exponential growth



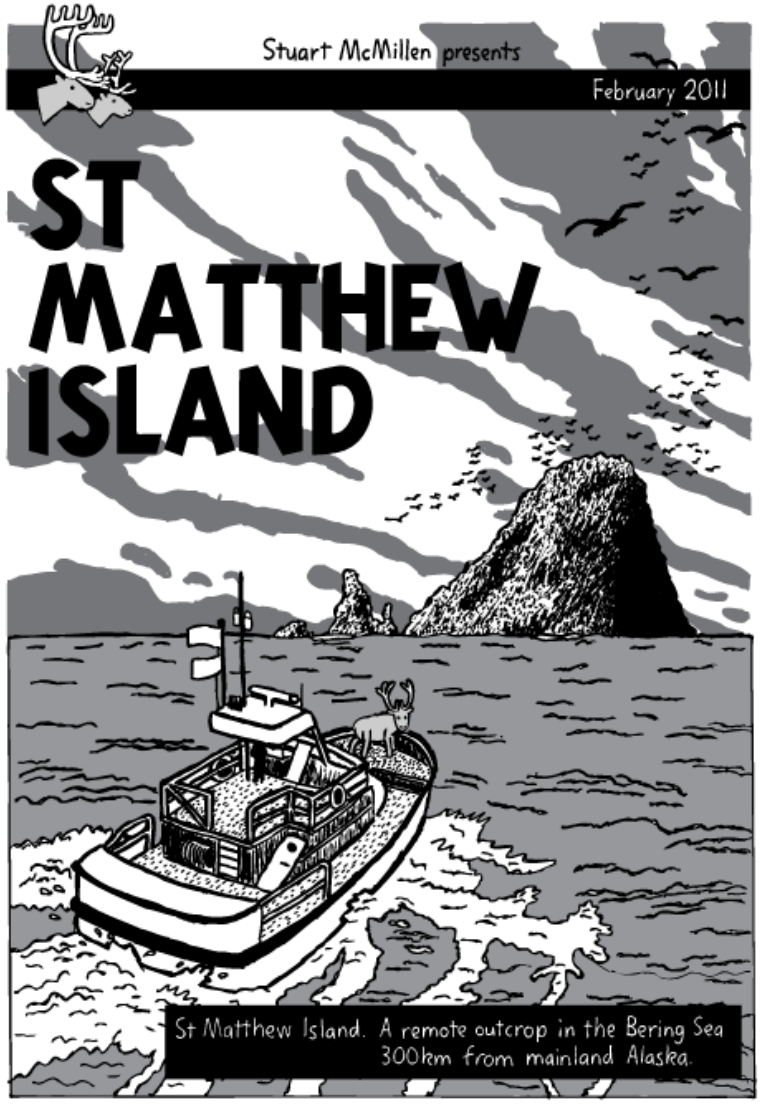
If this is correct, we could run out of space.

- Exponential fit: 1 m^2 of dry land per person in year 3620
- Logistic: 124.8m^2 per person at the limiting population.

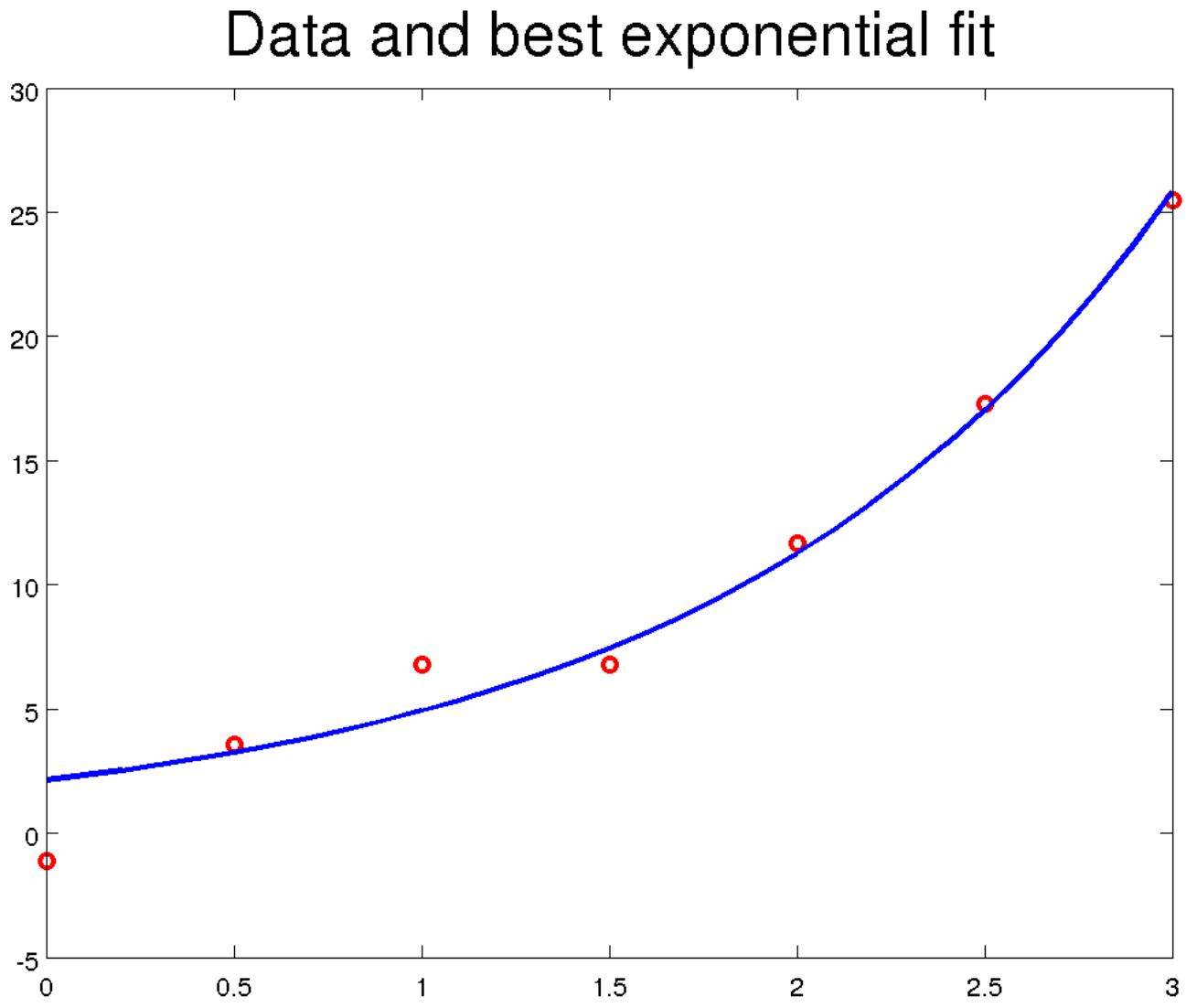


http://www.hydroville.org/CUBICBOXACTIVITY

Reindeer on St. Matthew's Island: A cautionary tale



How does this curve-fitting work?



A fundamental truth about human population.

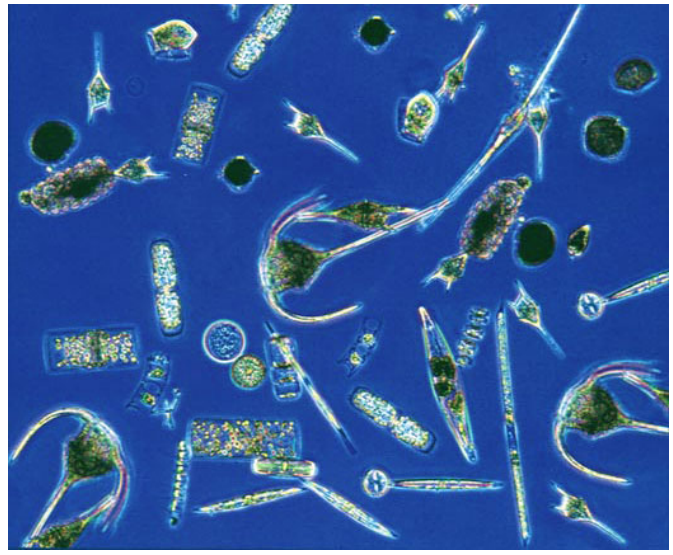
The future depends on the past only through the present.

Demographers predict and estimate from birth and death estimates.

Did the mathematical model give us insight on the question of overpopulation?

My Research: Phytoplankton population modeling

- "Plant- wanderer" – photosyntesizing organisms in water
- Agents of primary production – base of food web
- Depend on minerals: nitrogen, phosphorus, silica
- Sequester CO_2



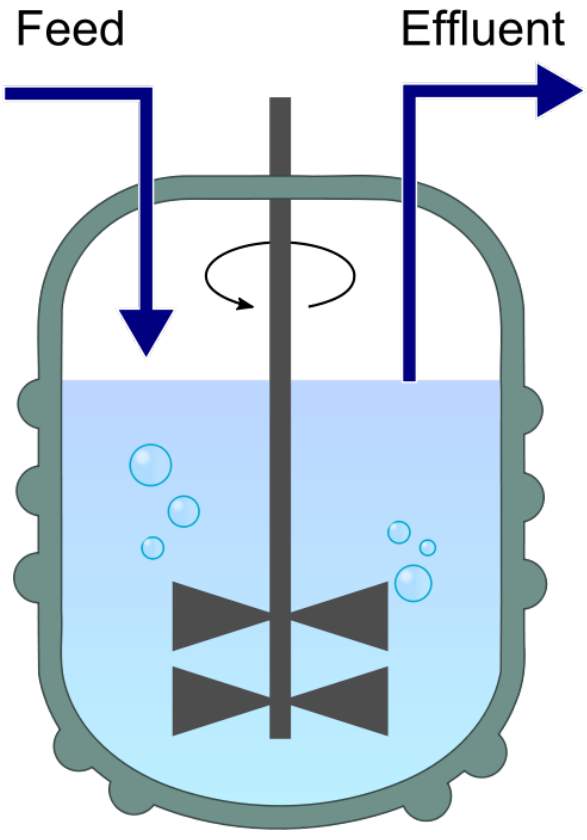
Credit `http://www.seos-project.eu/modules/oceancurrents/images/mt001a5_rd01.jpg`

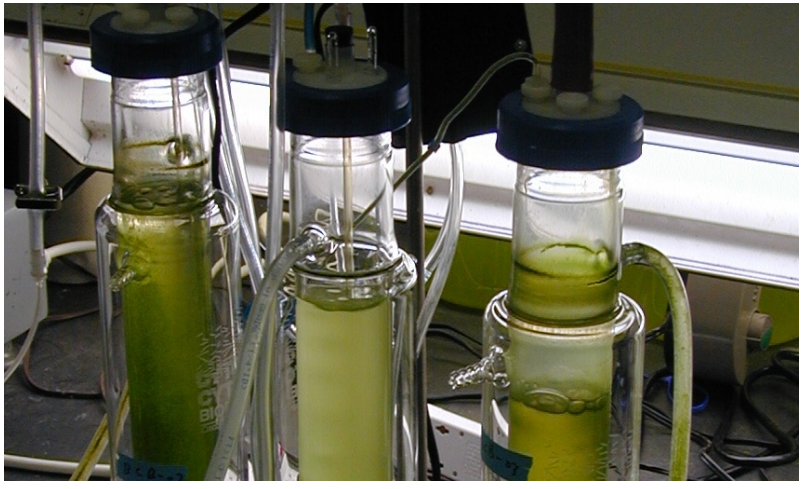
Harmful blooms can create a disgusting mess, kill fish, etc.

(All algae are plankton, but not plankton are algae...)

Credit `http://toxics.usgs.gov/highlights/algal_toxins/`

Experimental setup with a chemostat





Credit: `http://biology.mcgill.ca/faculty/fussmann/` for the chemostat photo.

The chemostat model is from WikiMedia Commons.

Mathematical Model

- $P_i(t)$ population density of phytoplankton, $i = 1, \ldots, n$
- $N_j(t)$ nutrient (substrate) concentration, $j = 1, \ldots, m$

$$\frac{dP_i}{dt} = \mu_i P_i - \nu P_i$$

$$\frac{dN_j}{dt} = \nu(N_{\text{in}} - N_j) - \sum_{i=1}^{n} Q_i \mu_i P_i$$

Liebig's Law of the Minimum/Essential nutrient

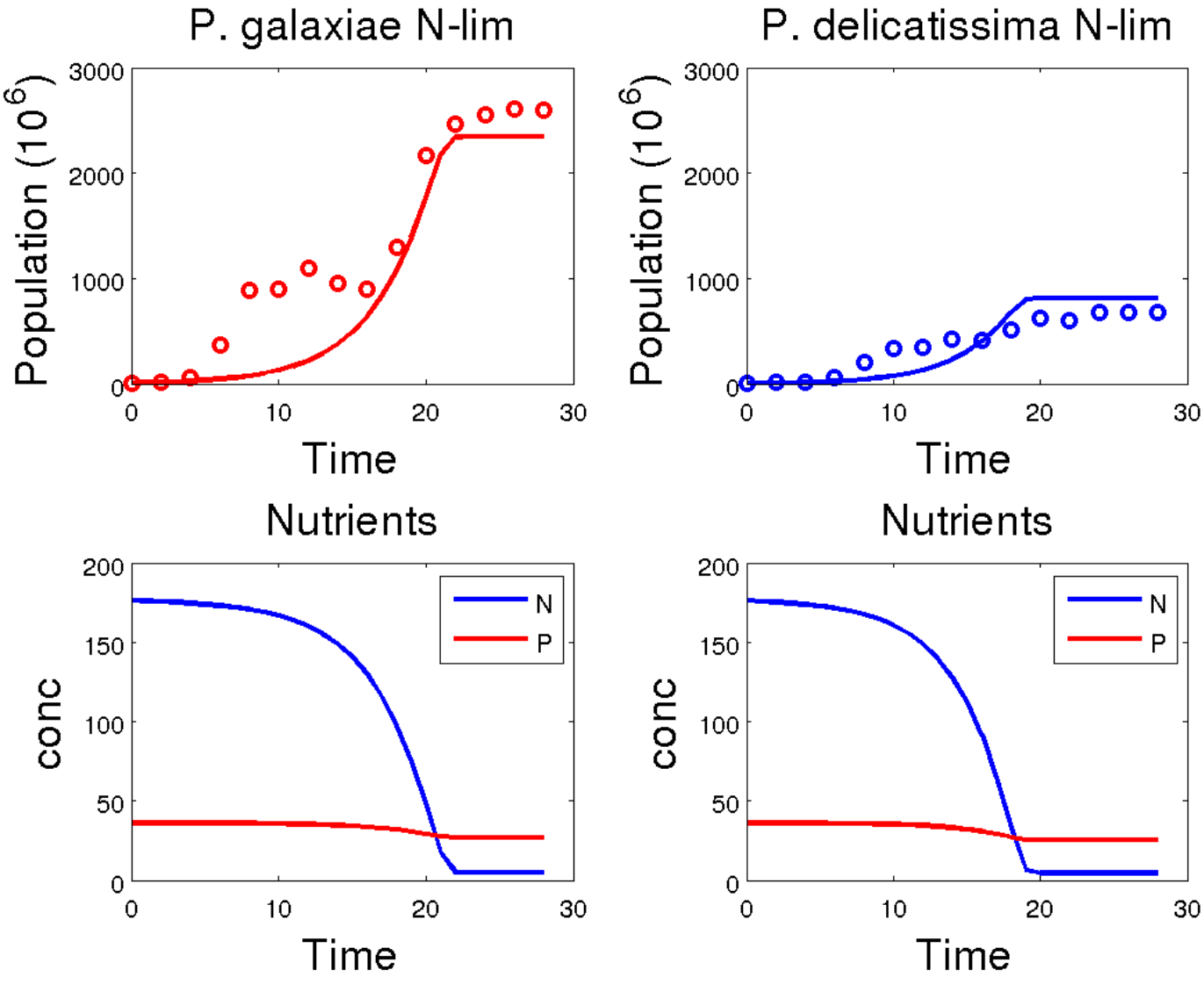
$$\mu_i = \tilde{\mu}_i \min_j \left\{ \frac{N_j}{N_j + k_{sj}} \right\}$$

P. delicatissima and P. galaxiae

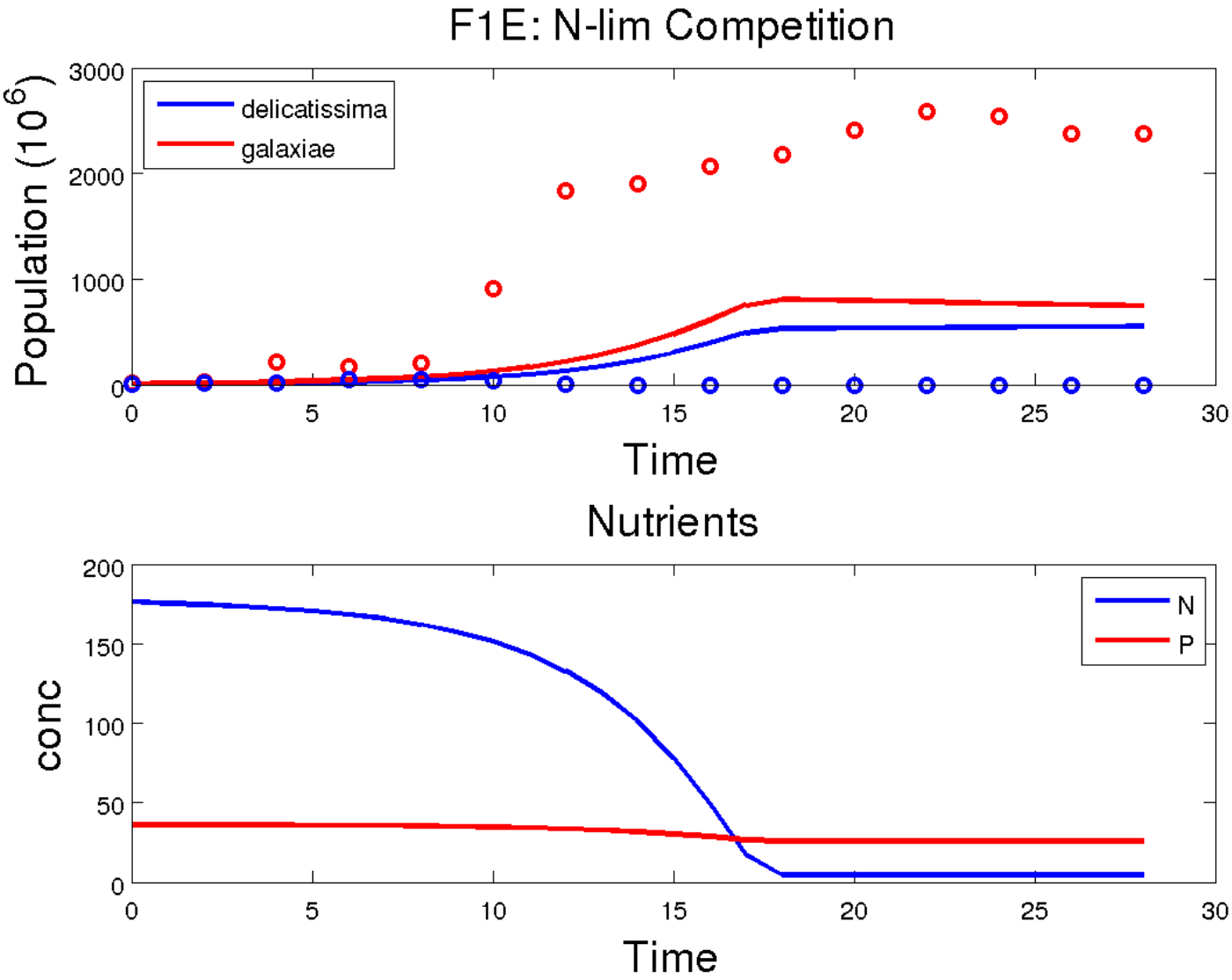
- Produce a neurotoxin that causes Amnesic Shellfish Poisoning
- Found in: Canada, US West Coast, Europe, Asia, New Zealand, Central and South America
- Frequently coexist (are found together) in nature

Credit: Phytoplankton Encyclopedia Project P. delicatissima

Data and model results – Nitrogen limitation



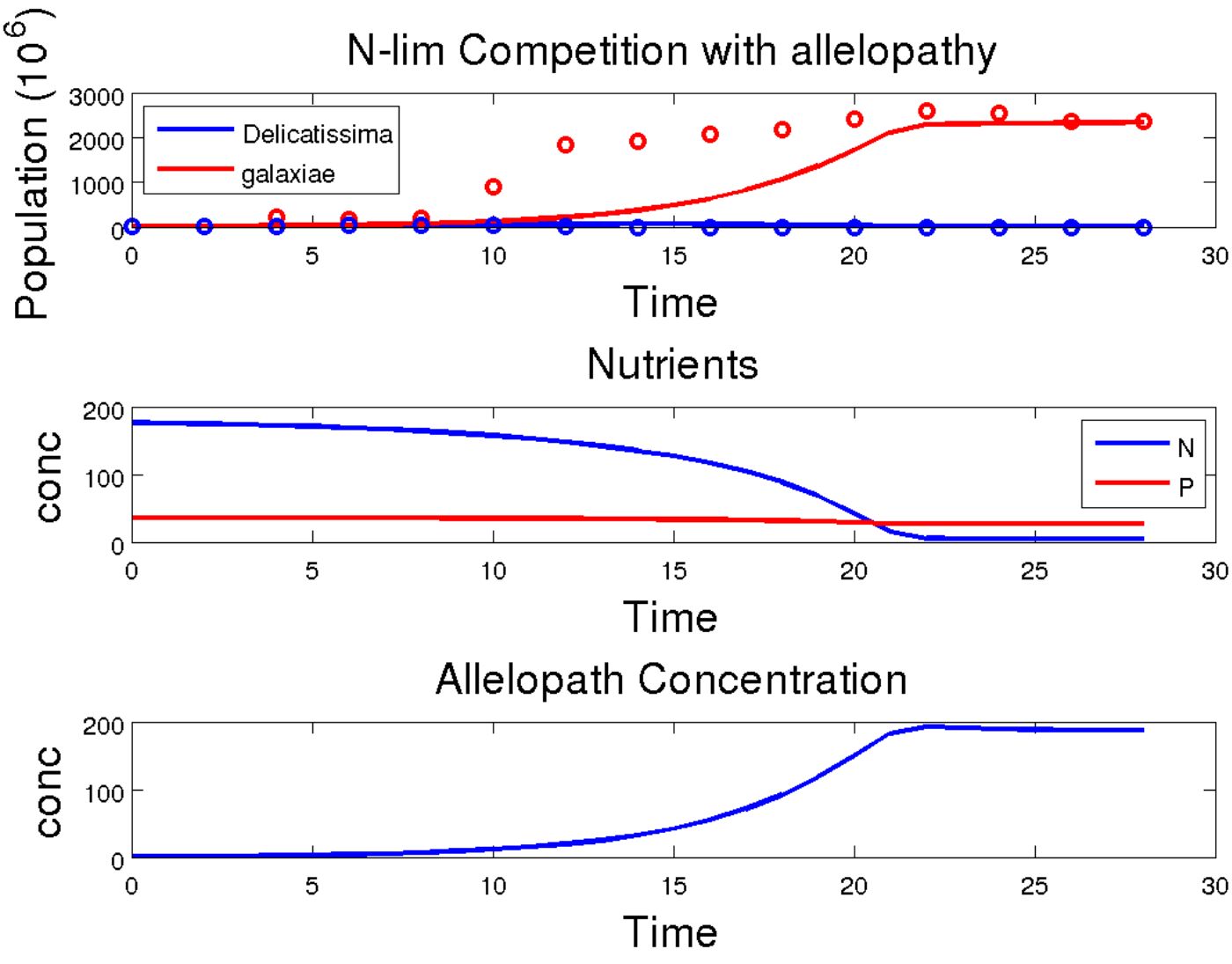
Species competition: data and model are not the same



Why? And what about coexistence?

- Allelopathy: chemical warfare by P. galaxiae
- Coexistence: differential sinking rates?

Model with allelopathy



This mathematical model and insight into the real world

- Good insight from the real world required!
- Math can tell us when we need more information.
- Interplay of insight: real world $\longleftrightarrow$ math.

Metacommunities: started with two REU students last summer

- "Patchy" ecosystem
- Interconnected: REU students helped set up computer model
- Linked by species dispersal
- Big question: connection to diversity in the environment?
- REU students: some initial experiments.

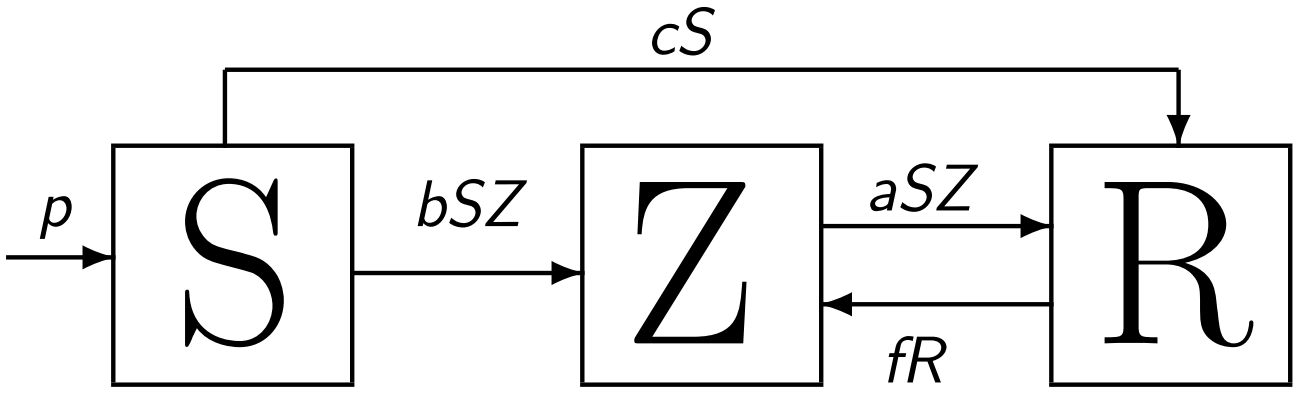
Photo credit: `http://www.cottenielab.org/2011/02/metacommunity-dynamics-in-prairie.html`

Best class project

Mathematical modeling of a zombie outbreak!

Population modeling based on Susceptible-Infected-Recovered (SIR) models from epidemiology

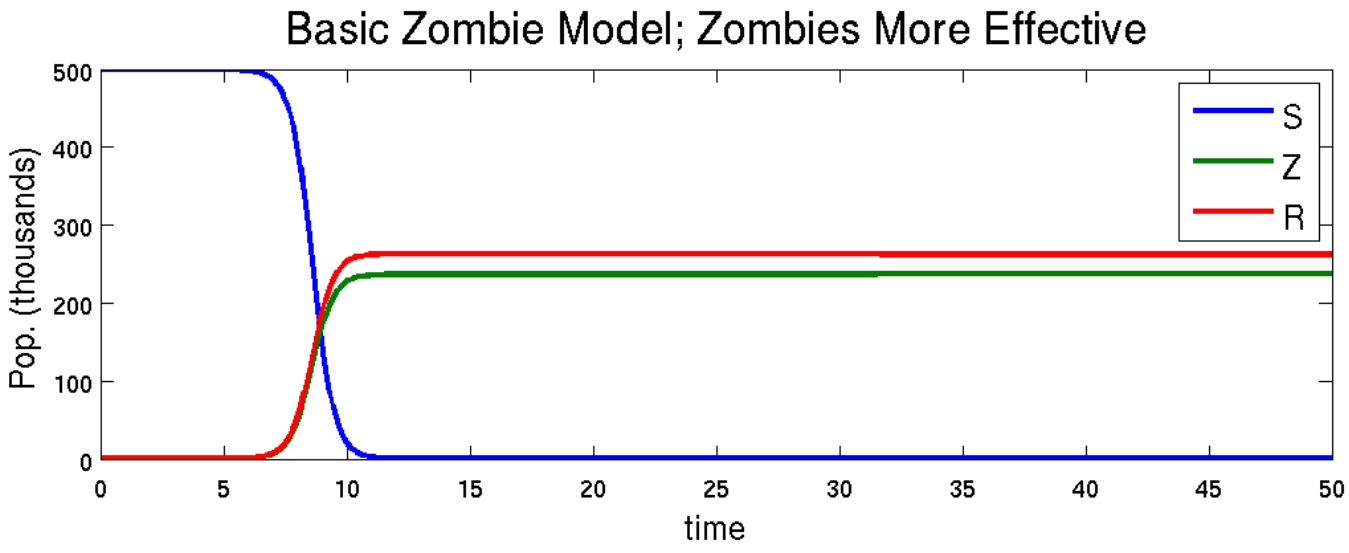
Basic zombie model

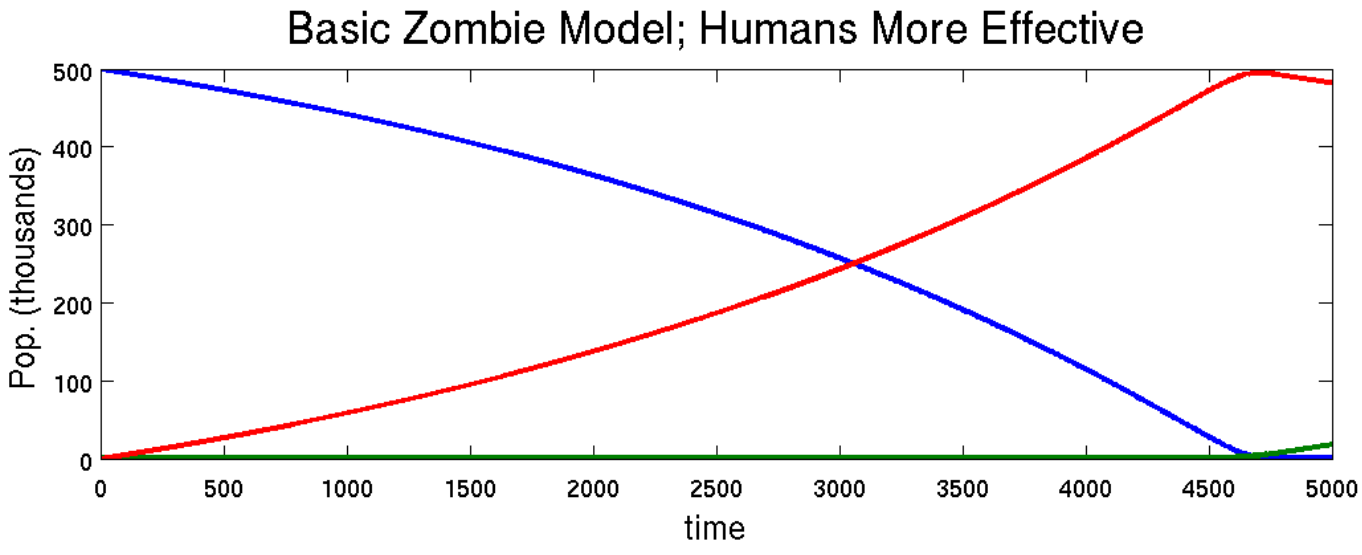


$$\frac{dS}{dt} = p - bSZ - cS$$
$$\frac{dZ}{dt} = bSZ - aSZ + fR$$
$$\frac{dR}{dt} = aSZ + cS - fr$$

Credit: Munz, et. al. When zombies attack!: Mathematical modelling of an outbreak of zombie infection

This model is **bad news** for humans





Challenge to students!

Can you come up with a realistic hypotheses for zombie outbreak and a corresponding mathematical model in which humans survive?

What do you need to get started? What can I offer?

To get started on one of these projects:

- ~~Differential calculus~~ ~~**or**~~ **willingness to learn**
- ~~Ideal: differential equations~~ **willingness to learn**
- ~~Wish list: computer programming~~ **willingness to learn**

What can I offer?

- Have an idea, need math? Let's talk!
- Have an idea, need a (math) mentor? Let's talk!
- I want to learn and try new things.

Favorite great general audience science sites

FYFD! Fluid Dynamics (Tumblr)

The Brain Scoop Videos (YouTube)
The Brain Scoop (Tumblr)

Thanks for listening!