



Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

References

# Research with Zombies

Jean Marie Linhart



[jmlinhart@math.tamu.edu](mailto:jmlinhart@math.tamu.edu)

2013

- 1 Students love them.
  - Humans vs. Zombies (HvZ) (HvZauthors [2012])
  - World War Z: ~\$500 million box office receipts worldwide (7/23/2013)
  - Zombies are worth \$5 billion in today's economy. (Ogg [2011])
- 2 Accessible to calculus students.
- 3 Room for creativity.
- 4 Lots of interesting math.





# Epidemiological models for a Zombie outbreak

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

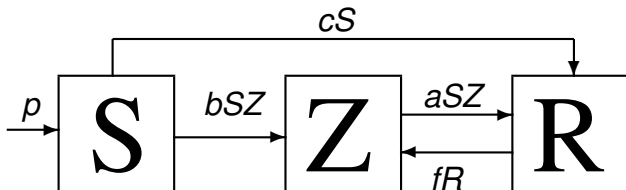
Stochastic  
Results

More Zombie  
Mathematics

References

Zombie disease models were popularized in 2009 with the publication of “When Zombies Attack” in *Infectious Disease Modelling Research Progress*. (Munz et al. [2009])

- **S**: Susceptibles, uninfected humans
- **Z**: Zombies (normally  $I$  for infected)
- **R**: Removed, the dead



$$\frac{dS}{dt} = p - bSZ - cS$$

$$\frac{dZ}{dt} = bSZ - aSZ + fR$$

$$\frac{dR}{dt} = aSZ - fR + cS$$

$a, b, c, p, f$  are all constants (parameters) in the model



# Model Results: doomsday

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

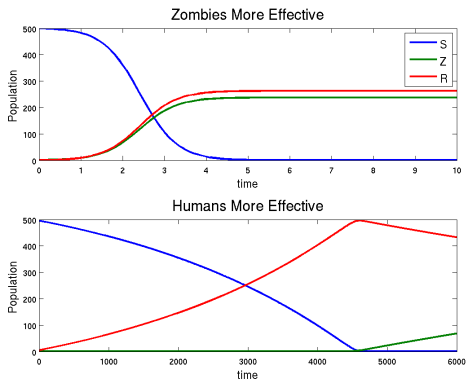
References

Parameter values  $p = 0.0001$ ,  $f = 0.0001$ ,  $c = 0.0001$

Both started with 500 humans, 1 zombie.

First plot  $a = 0.005 < b = 0.0095$ ; zombies more effective

Second plot  $a = 0.01 > b = 0.008$ ; humans more effective





# Zombie Mathematics

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

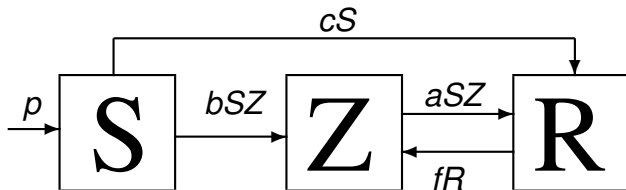
Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

References

- Mathematical modeling/Epidemiological modeling
- Applications of ordinary differential equations (ODEs)
- Autonomous differential equations
- Equilibria
- Stability
- Numerical methods for solving ODEs
- Use of a computer algebra system
- Reality into mathematics



What do you think? Is this model correct?



# Criticisms/Changes

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

References

- Shouldn't dead mean **dead**?
- Zombies should decay
- Better population model
- Necromancers
- Element of uncertainty





# Create a stochastic model

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

References

**stochastic** (adj.) involving chance or probability:  
probabilistic (Merriam-Webster.com [2013])

$$\frac{dS}{dt} = pS - bSZ - cS$$

$$\frac{dZ}{dt} = bSZ - aSZ + fR$$

$$\frac{dR}{dt} = aSZ - fR + cS$$

Break model into events:

human births	$pS$	humans $\rightarrow$ zombies	$bSZ$
human deaths	$cS$	zombie deaths	$aSZ$
		dead $\rightarrow$ zombies	$fR$



# Markov Process

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

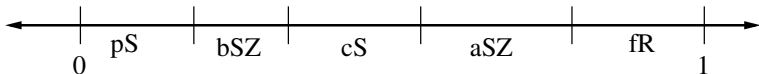
Stochastic  
Results

More Zombie  
Mathematics

References

All events:  $T(S, Z, R) = pS + bSZ + cS + aSZ + fR$

- Choose a uniform random value between 0 and 1.
- What happens is determined by where it falls in the interval.



- The time to next event is exponentially distributed.
- To determine it choose  $u$  a second uniform random value between 0 and 1 and calculate

$$\Delta t = -\frac{\log(u)}{T(S, Z, R)}$$



# Stochastic Realizations

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

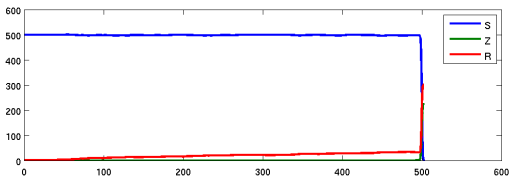
References

## Parameter values

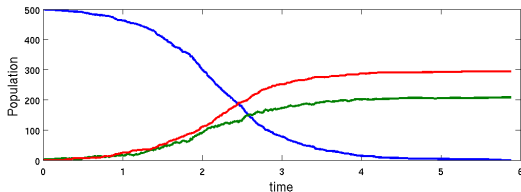
$p = 0.0001$ ,  $f = 0.0001$ ,  $c = 0.0001$ ,  $a = 0.005$ ,  $b = 0.0095$

Started with 500 humans, 1 zombie.

Stochastic Zombie model



Second realization



- Stochastic processes
- Markov chains
- Mean, standard deviation
- Probability distributions

What will your students come up with?



Thanks for coming and thanks for listening!



# References

Research with  
Zombies

J. M. Linhart

Introduction

Basic Model

Basic model  
results

Zombie  
Mathematics

Criticisms

Stochastic  
model

Markov  
Process

Stochastic  
Results

More Zombie  
Mathematics

References

Shaun of the dead, 2004. URL <http://www.imdb.com/title/tt0365748/combined>. Edgar Wright (director).

Kelli Bordner and Noah Bordner. *Zombieville*. MikaMobile, 2010. URL <http://www.zombievilleusa.com/>. Videogame for iOS.

Leonardo Finocci. Friendly zombie. WWW, 2010. URL <http://leonardofinocchi.deviantart.com/art/Friendly-Zombie-151356204?q=&qo=>.

HvZauthors. Humans vs. zombies (hvz), 2012. URL <http://humansvszombies.org/>.

Merriam-Webster.com. "stochastic". Web, 2013. URL <http://www.merriam-webster.com/dictionary/stochastic>.

P. Munz, I. Hudea, J. Imad, and R. J. Smith? When zombies attack! Mathematical modeling of an outbreak of zombie infection. In J. M. Tchuente and C. Chiyaka, editors, *Infectious Disease Modelling Research Progress*, pages 133–150. July 2009. ISBN 978-1-60741-347-9. URL <http://mysite.science.uottawa.ca/rsmith43/Zombies.pdf>.

John C. Ogg. *Zombies worth over \$5 billion to economy, October 2011*. URL [http://www.msnbc.msn.com/id/45079546/ns/business-stocks\\_and\\_economy/t/zombies-worth-over-billion-economy](http://www.msnbc.msn.com/id/45079546/ns/business-stocks_and_economy/t/zombies-worth-over-billion-economy).